Decidability border for Petri nets with data: wqo dichotomy conjecture

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Outline

- (un)ordered data Petri nets
- standard decision problems
- Petri nets with homogeneous data
- undecidability
- decidability via wqo

unordered data Petri nets

- data domain $(\mathbb{N}, =)$

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arbitrary countable infinite set

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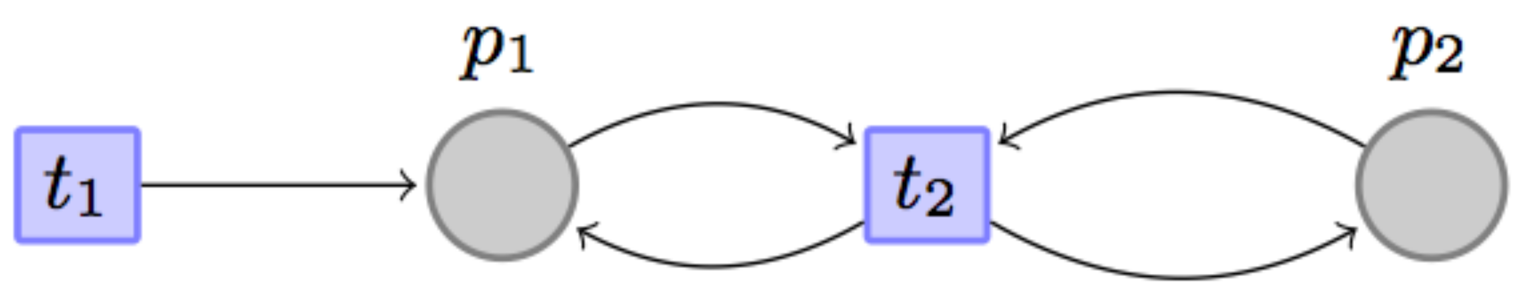

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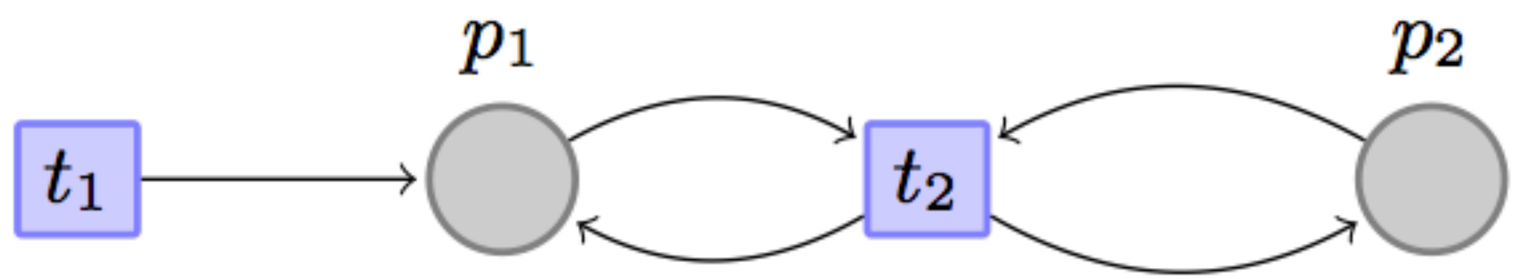

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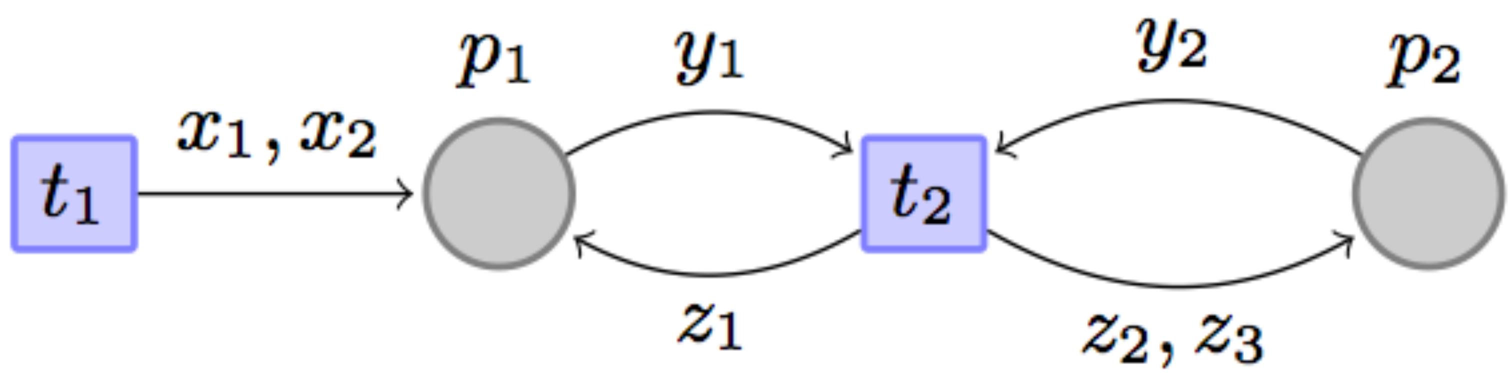

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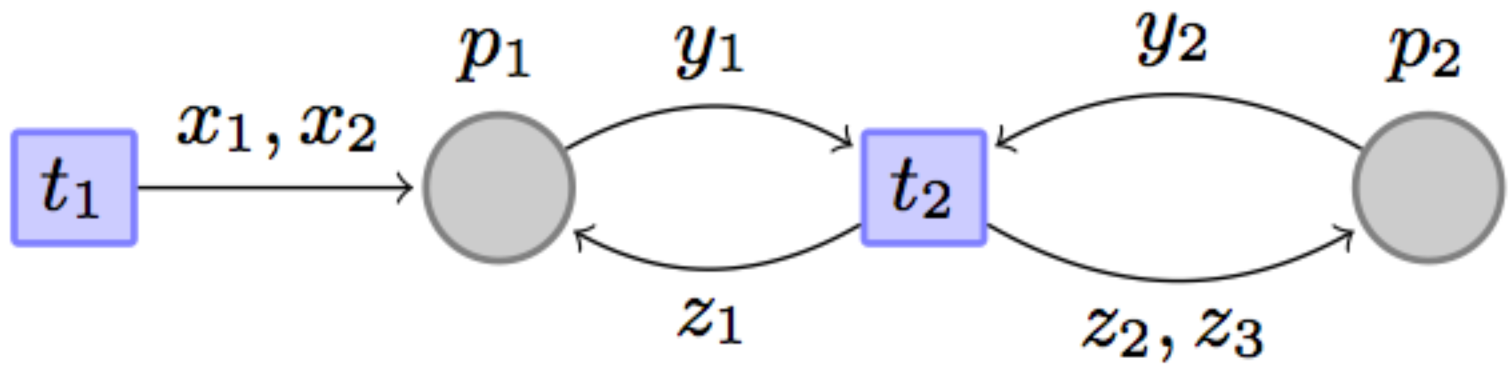

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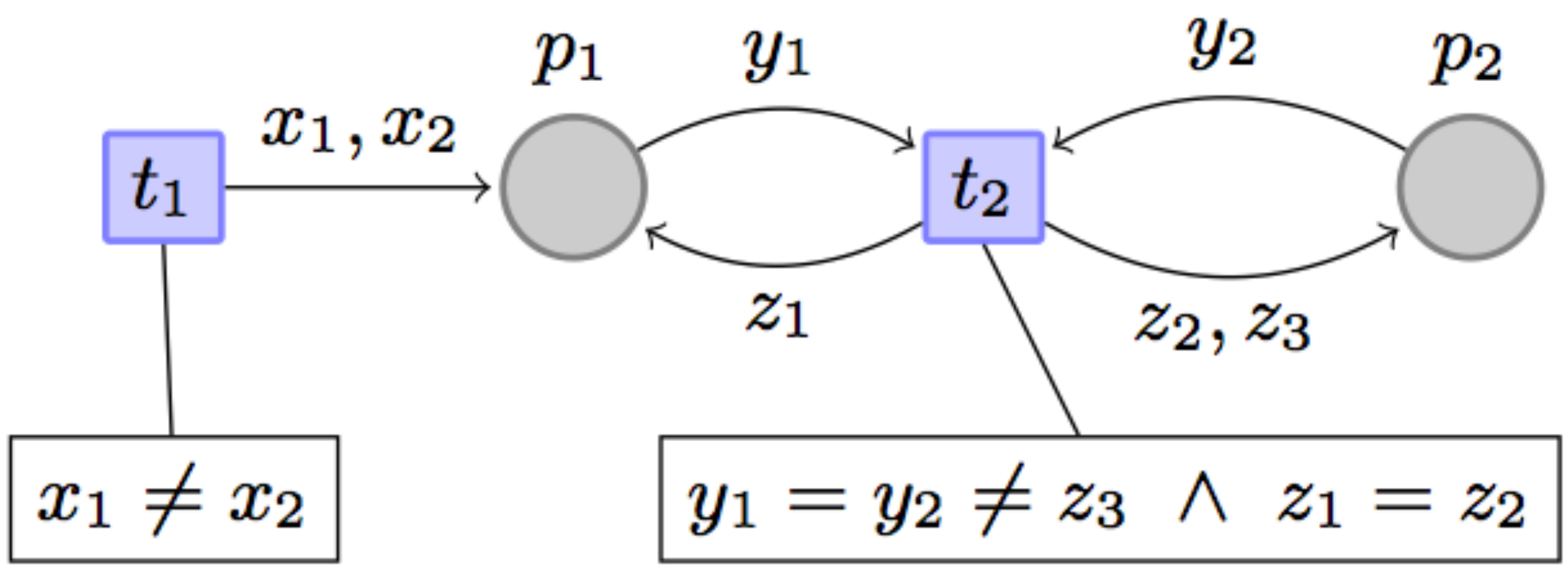

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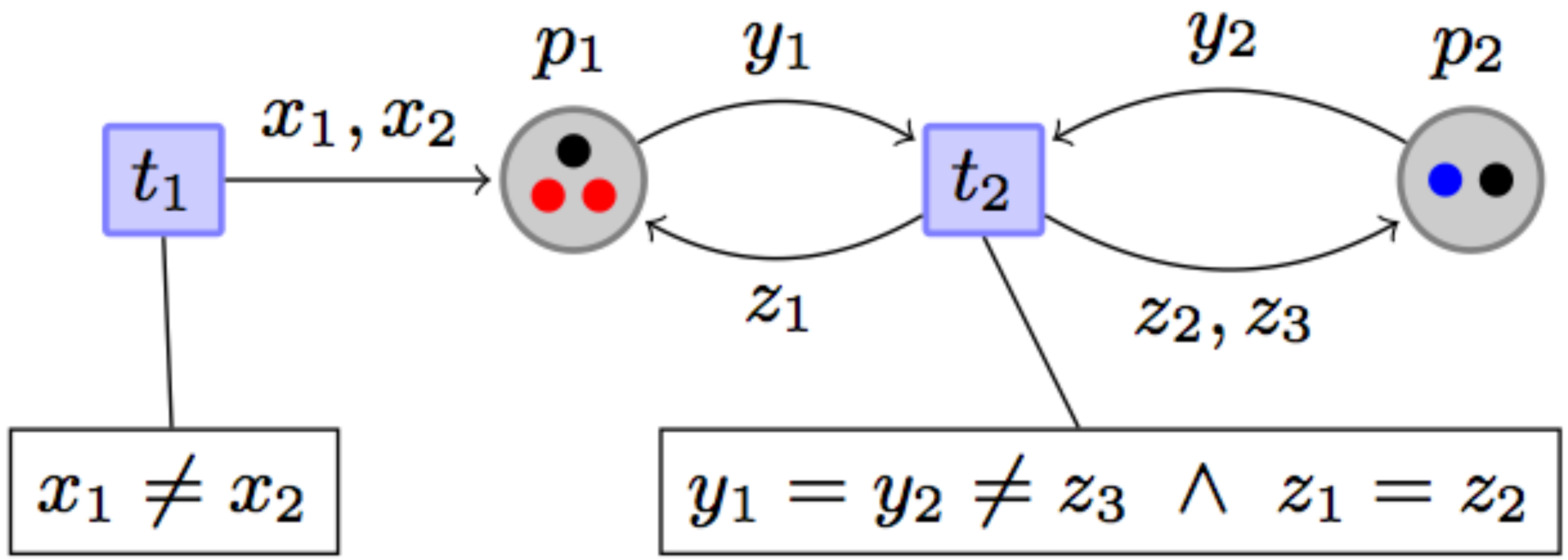

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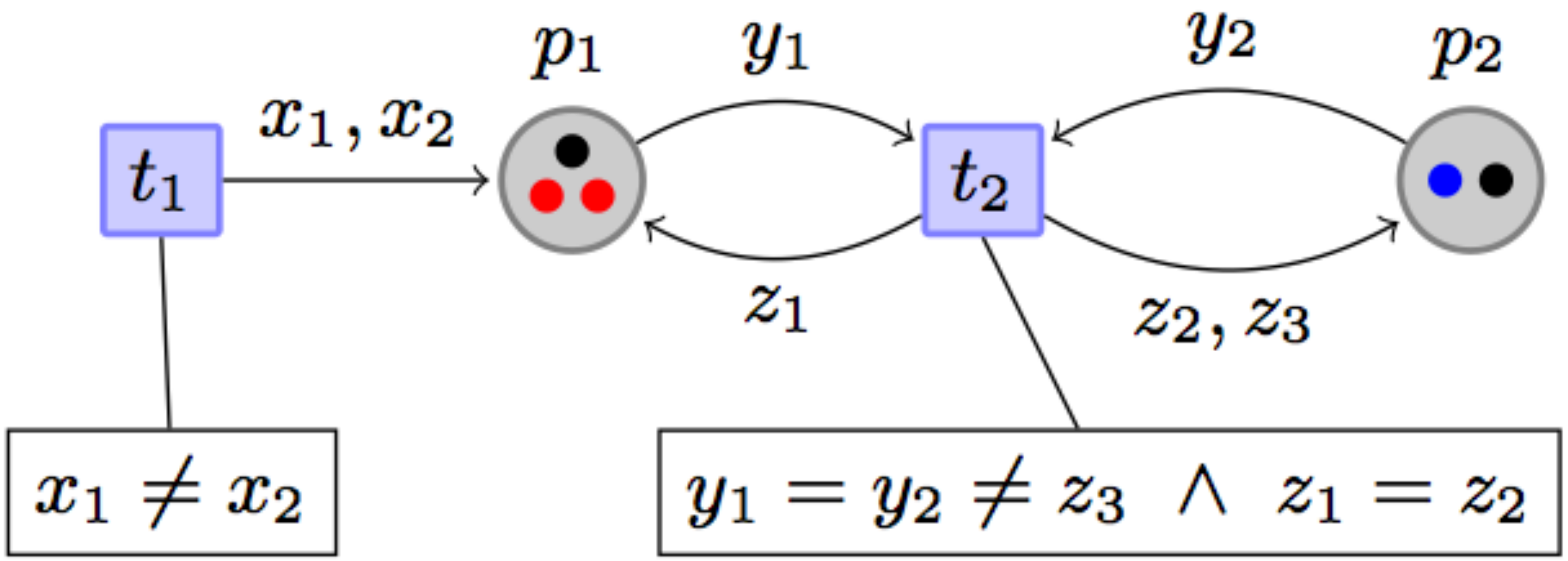

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- configurations: $M(\mathrm{P} \times \mathbb{N})$

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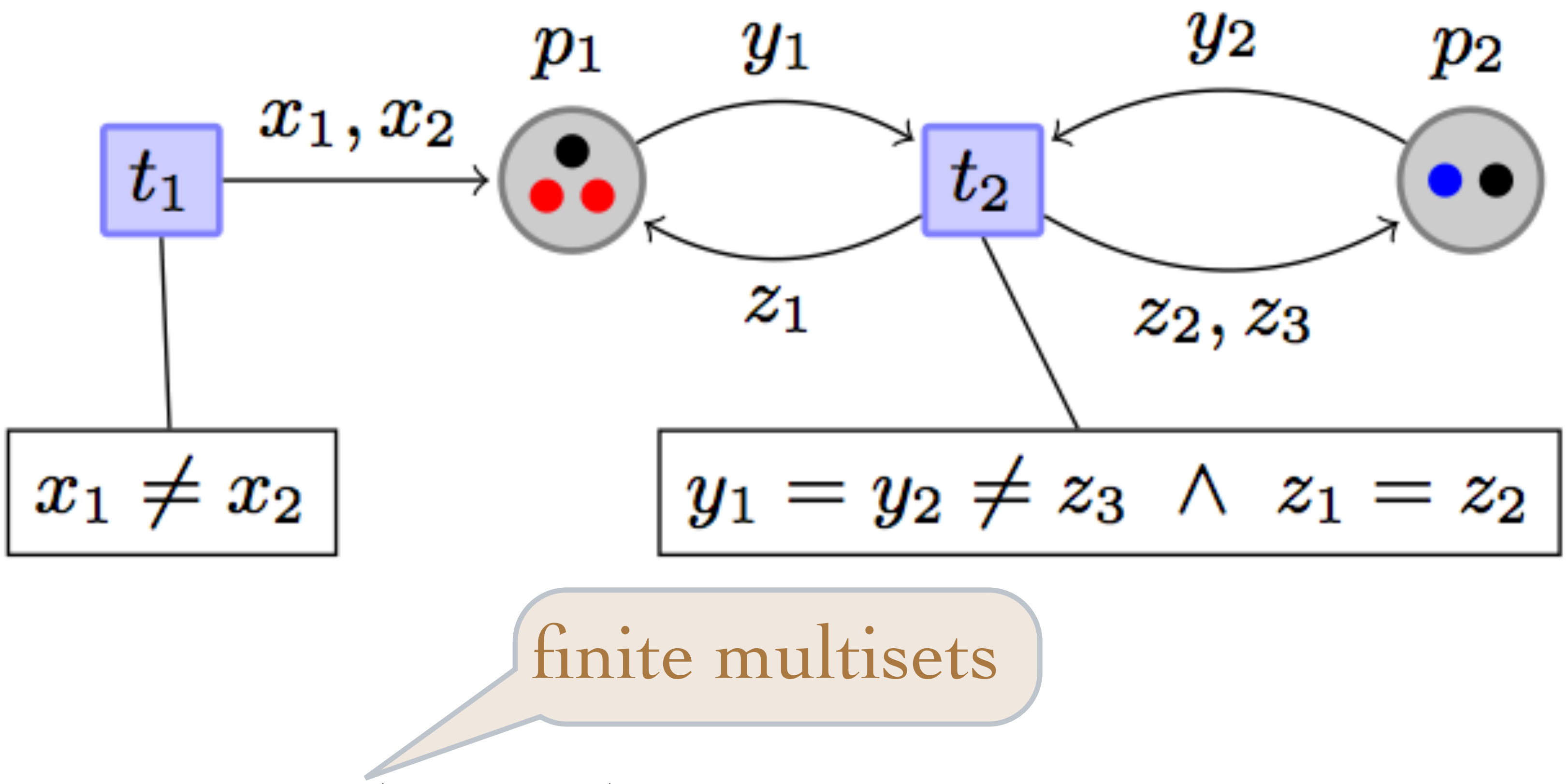

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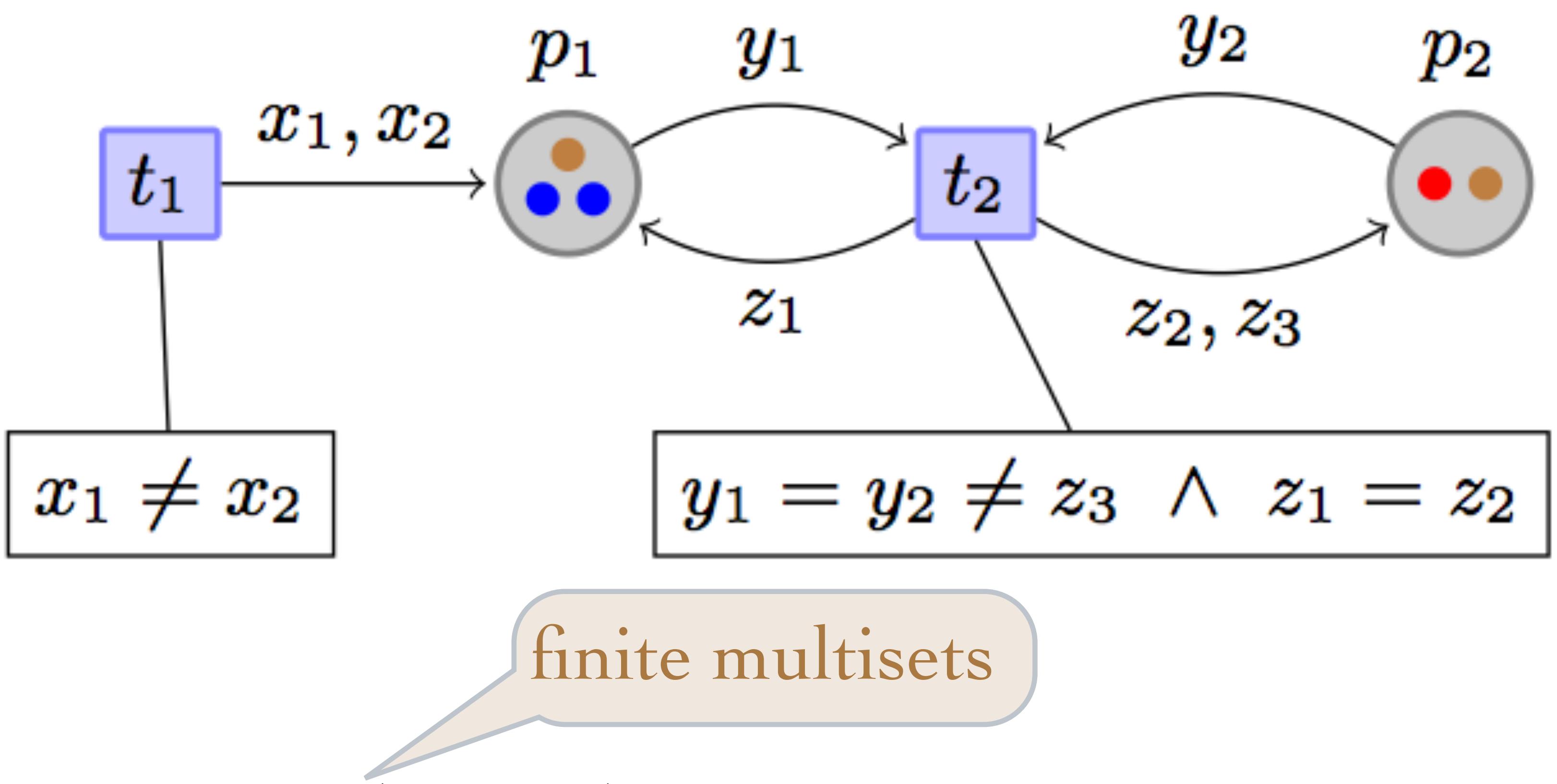

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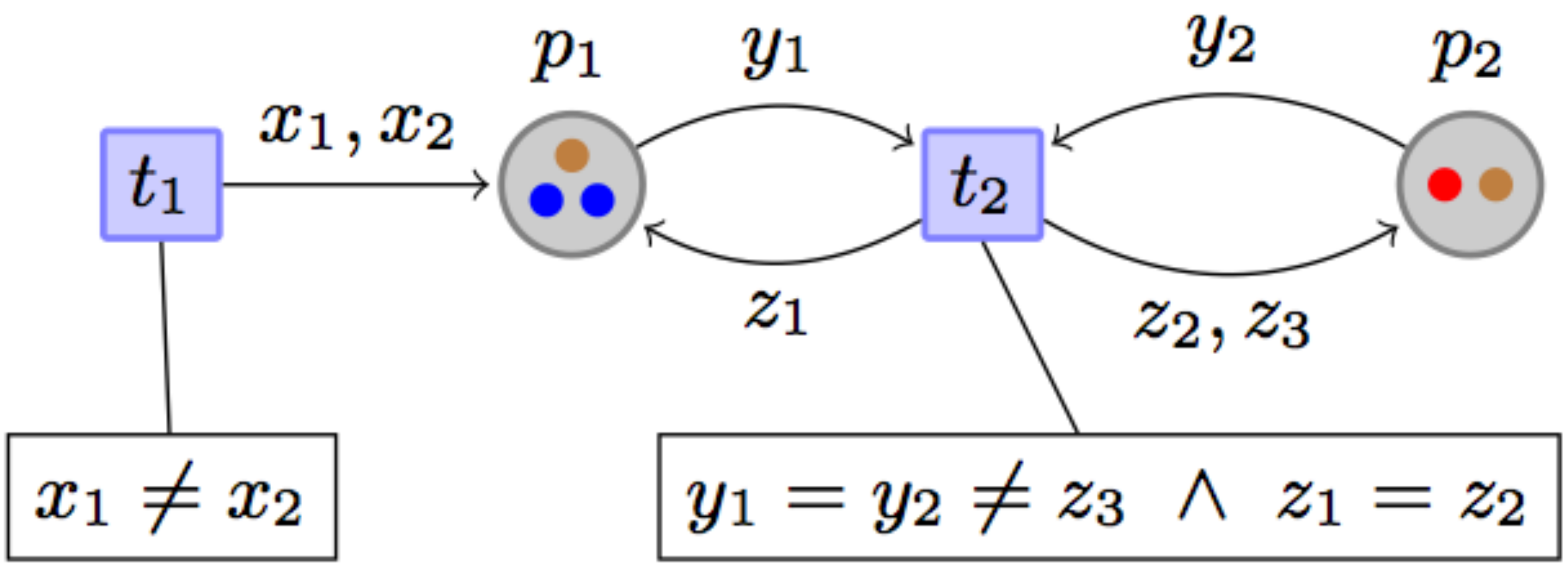

finite multisets

- configurations: $M(\mathrm{P} \times \mathbb{N})$
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- configurations up to data automorphism: $M(M(\mathrm{P}))$

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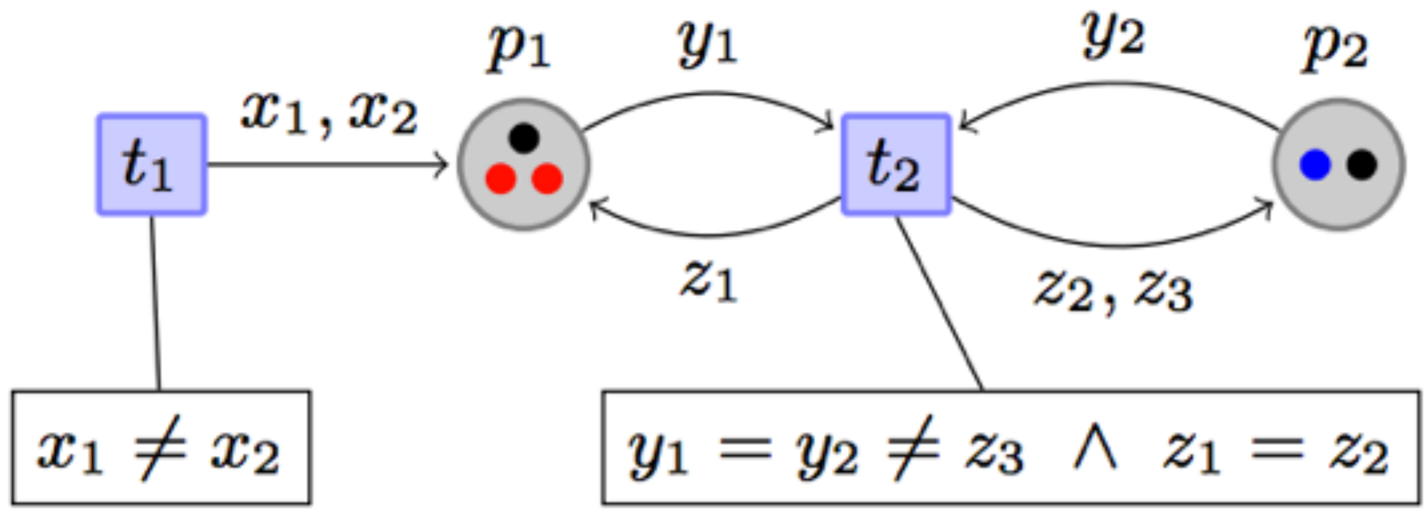
For $\mathcal{P}$ and $\mathcal{T}$ finite, this is essentially classical Petri nets.

semantics via multiset rewriting



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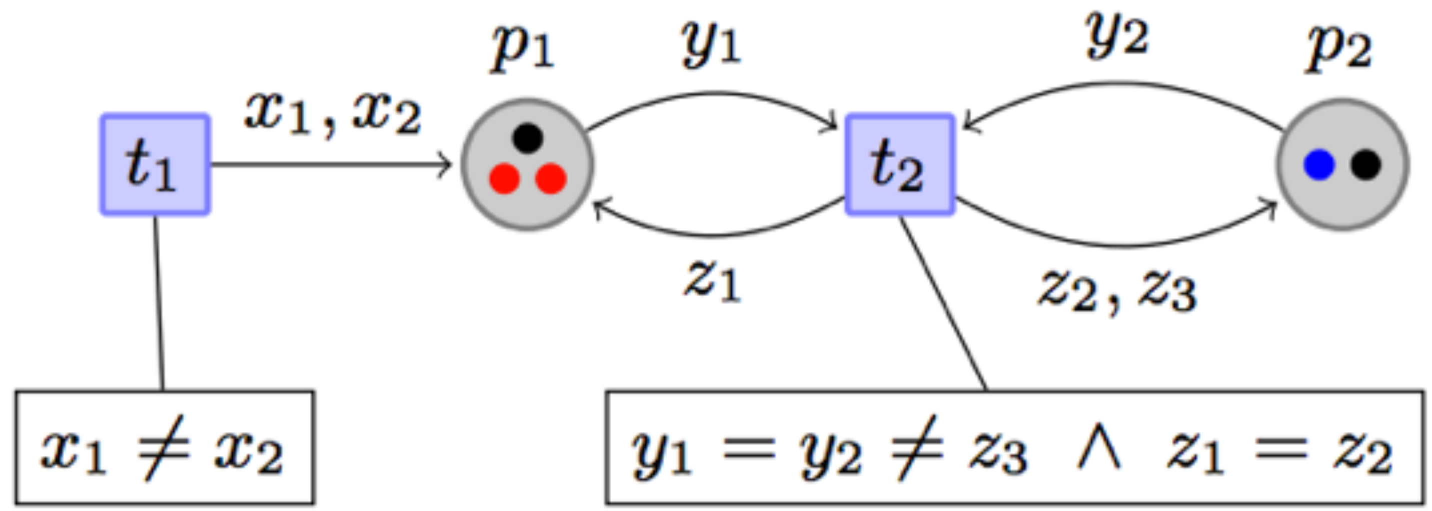

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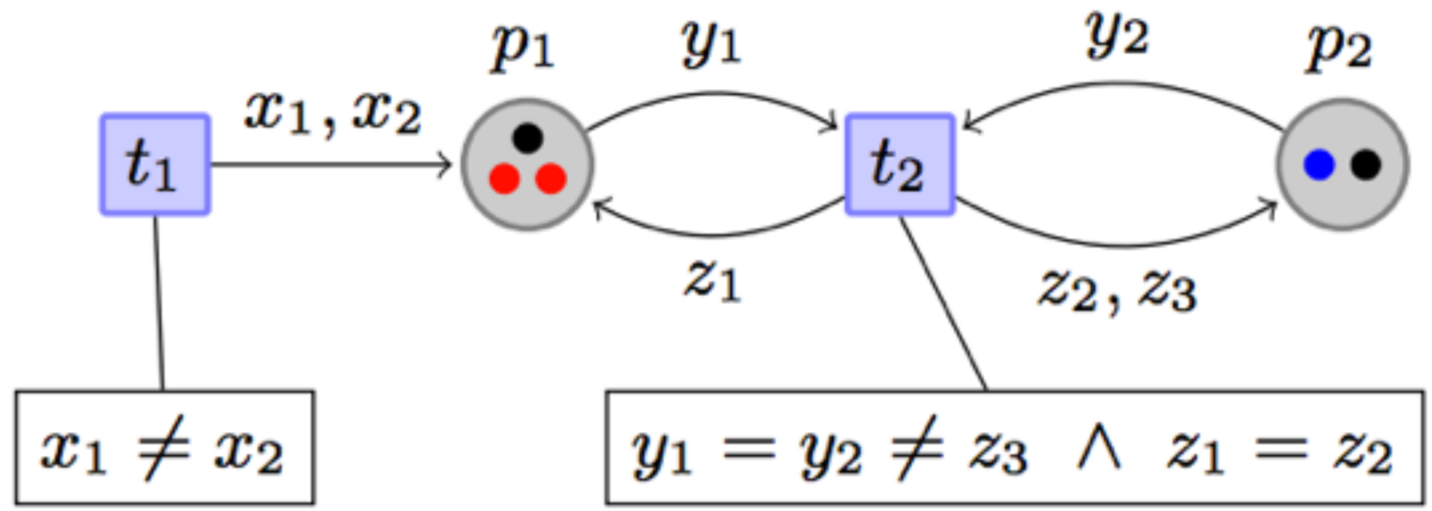

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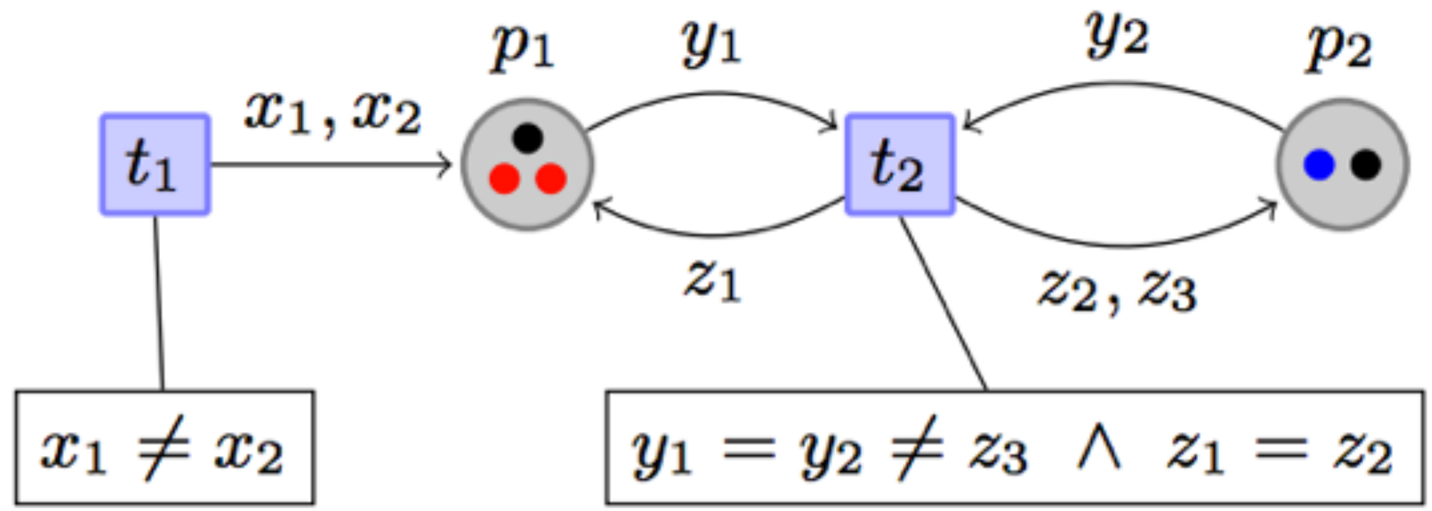

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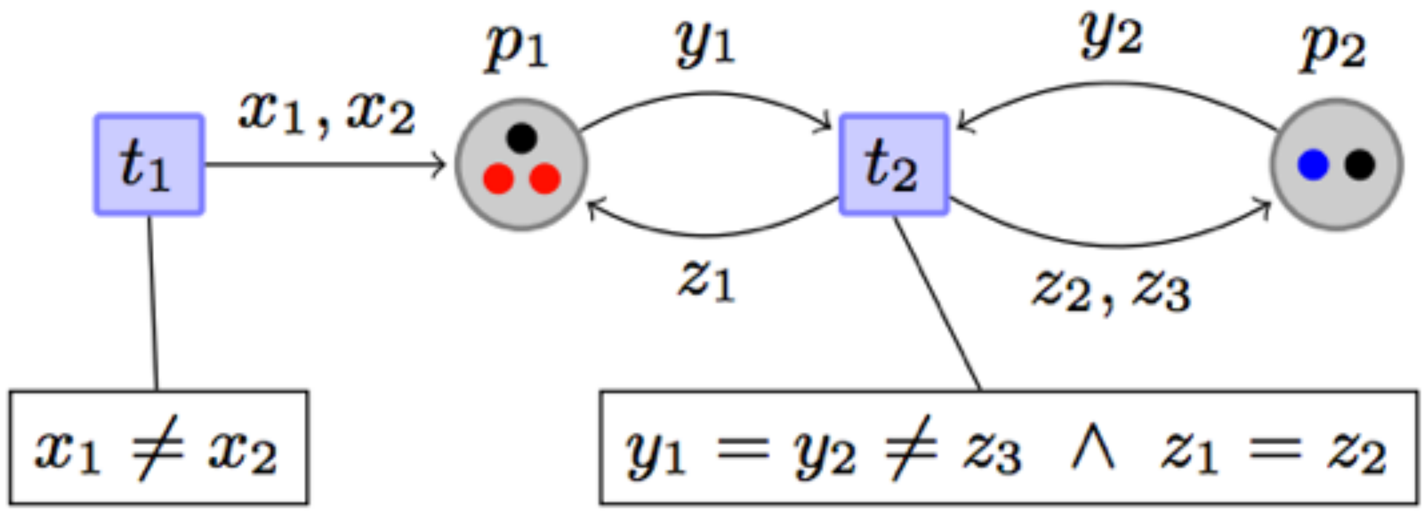

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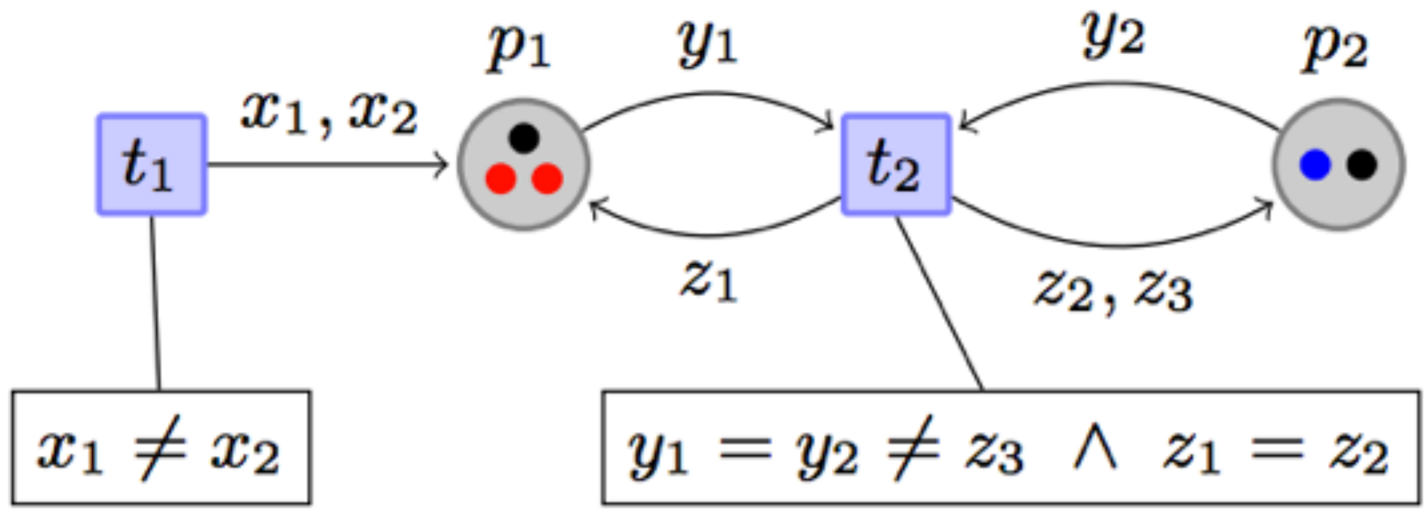

Petri nets, where instead of finite sets of places and transitions, infinite ones but definable in first-order logic; this follows the lines of [Bojańczyk, Klin, L.'14]

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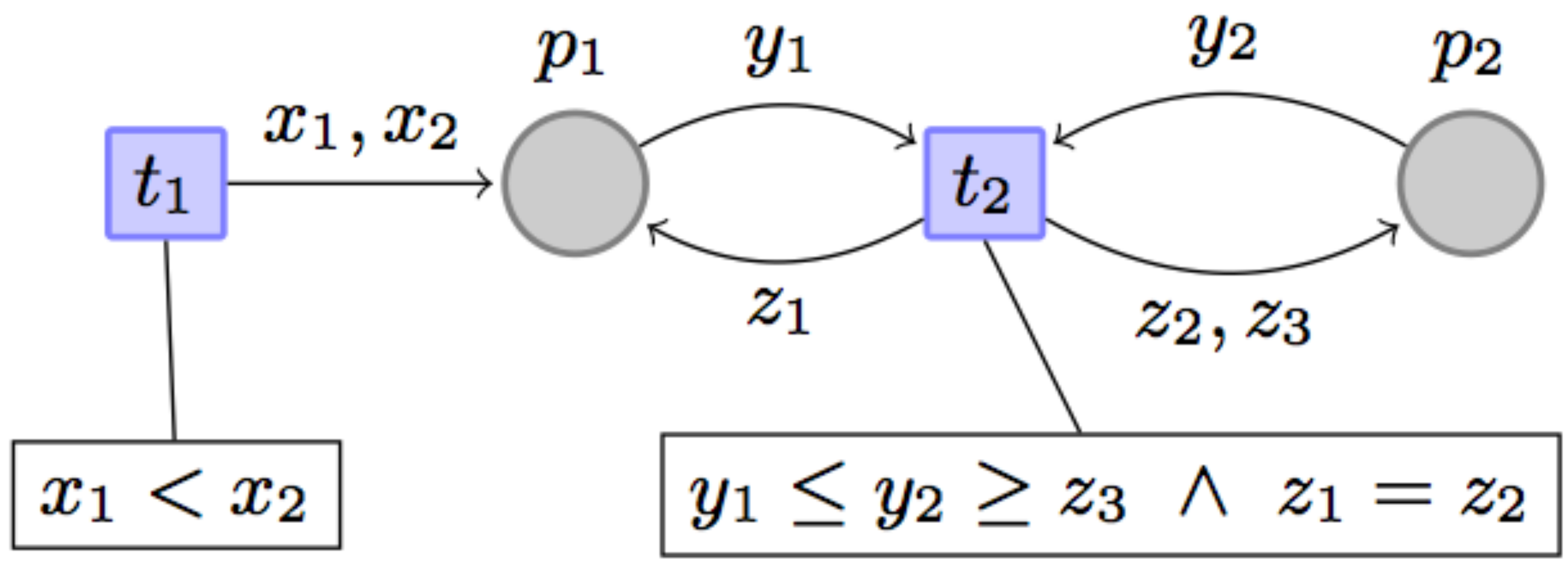

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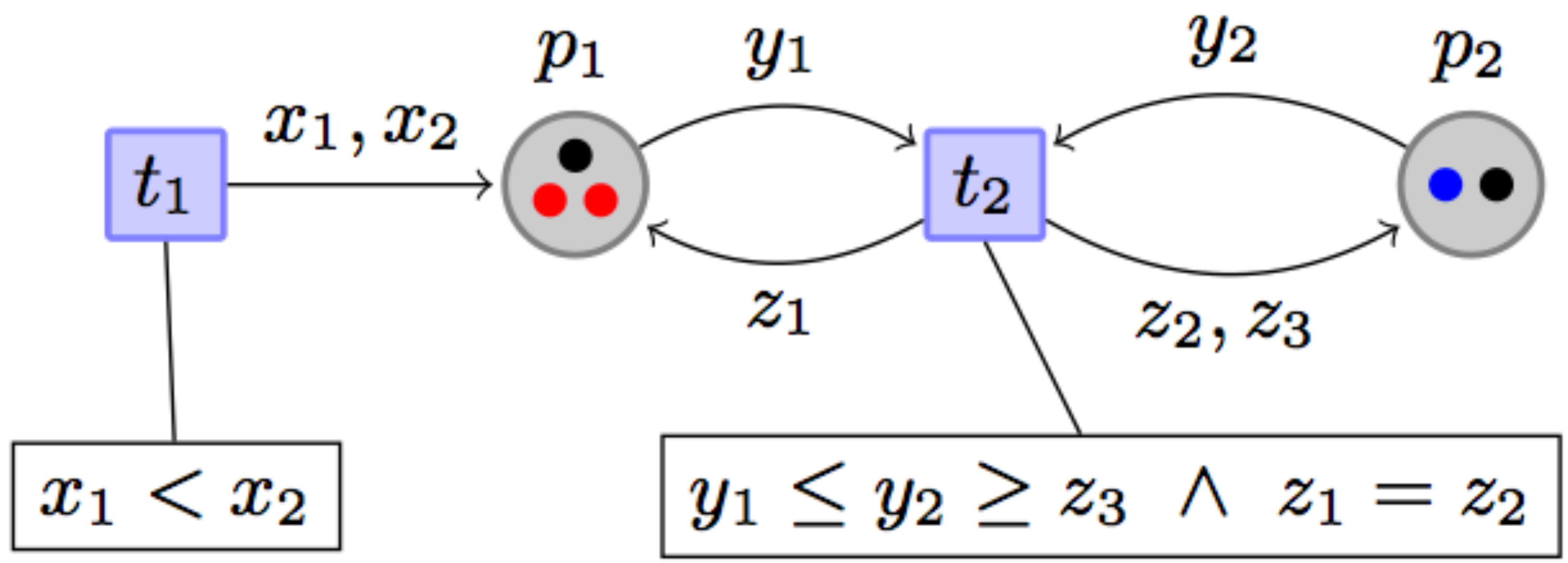

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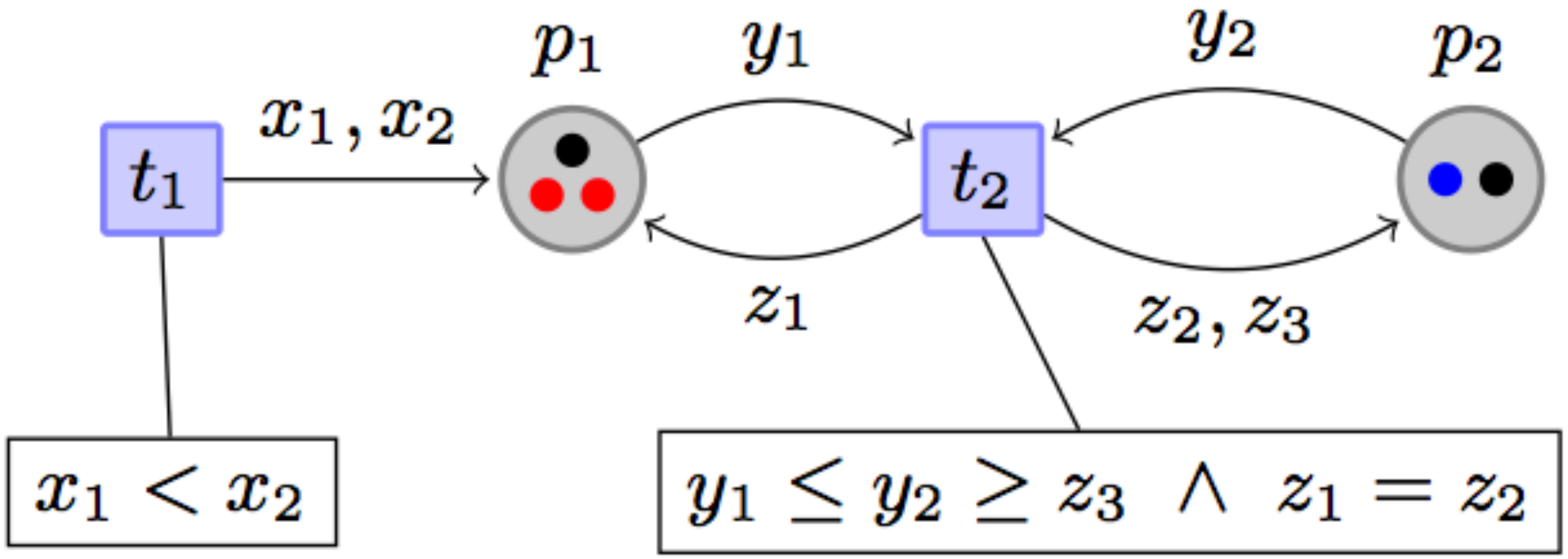

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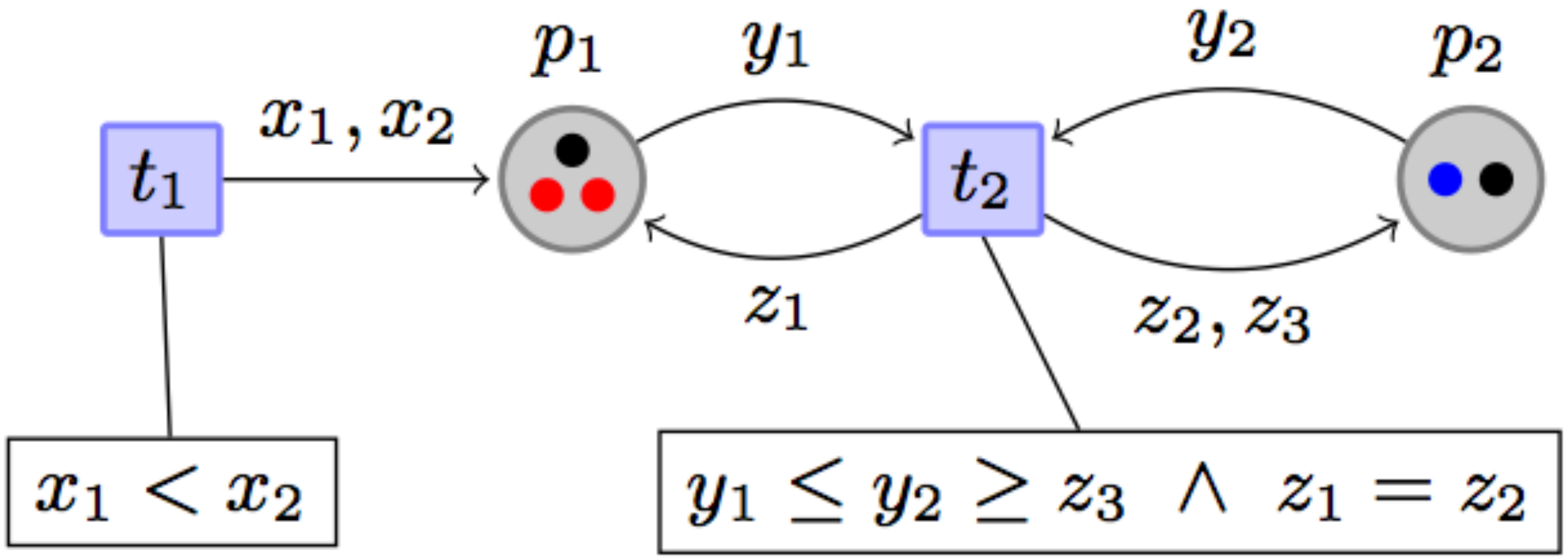

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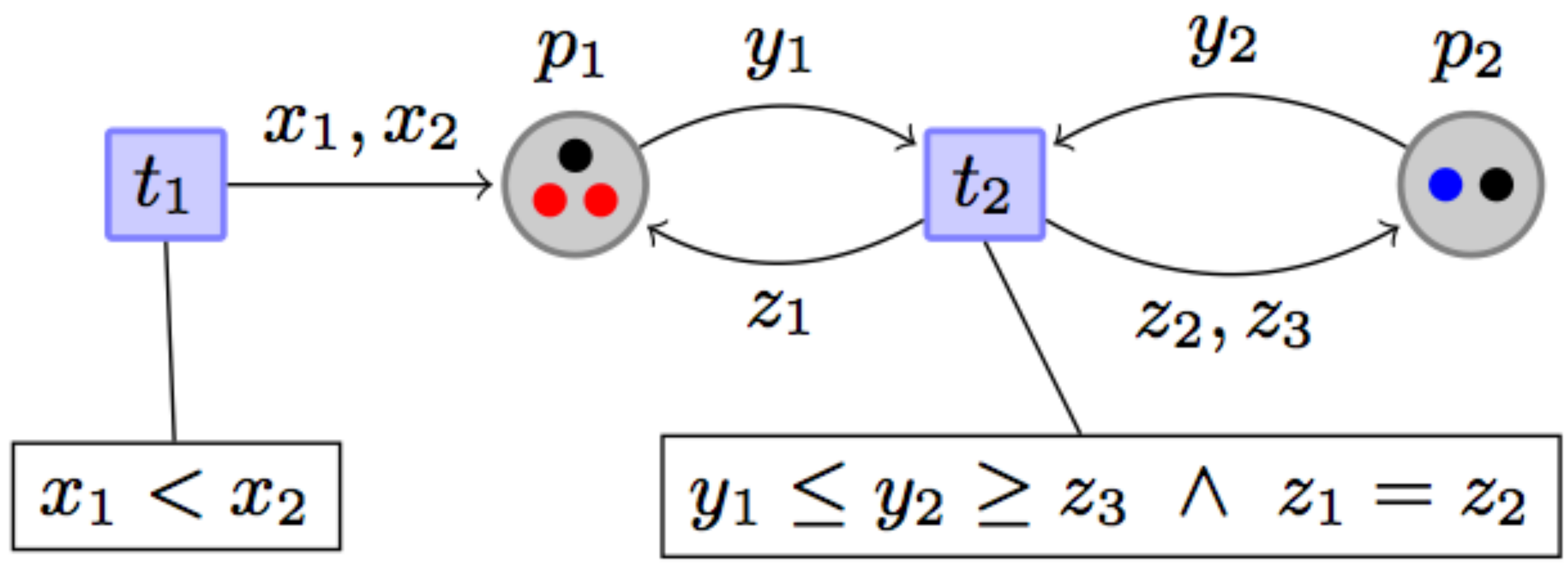

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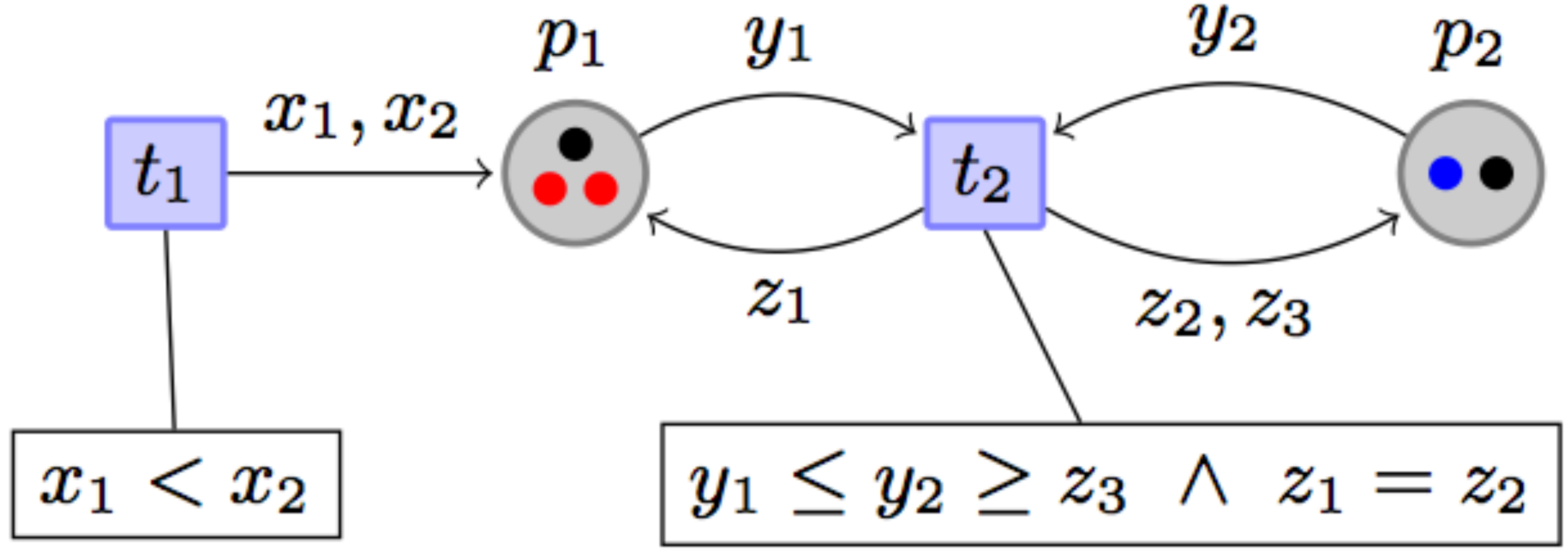

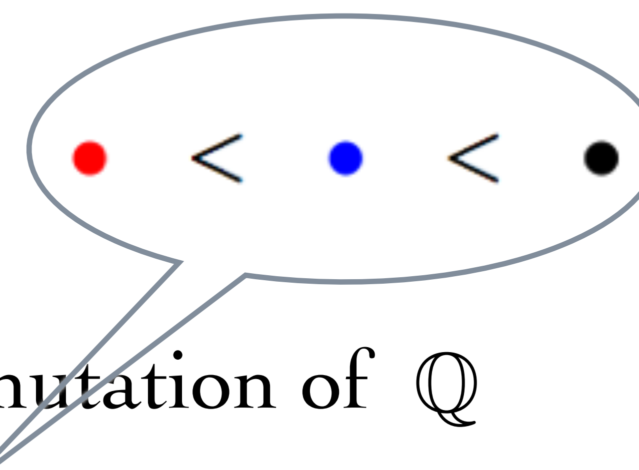

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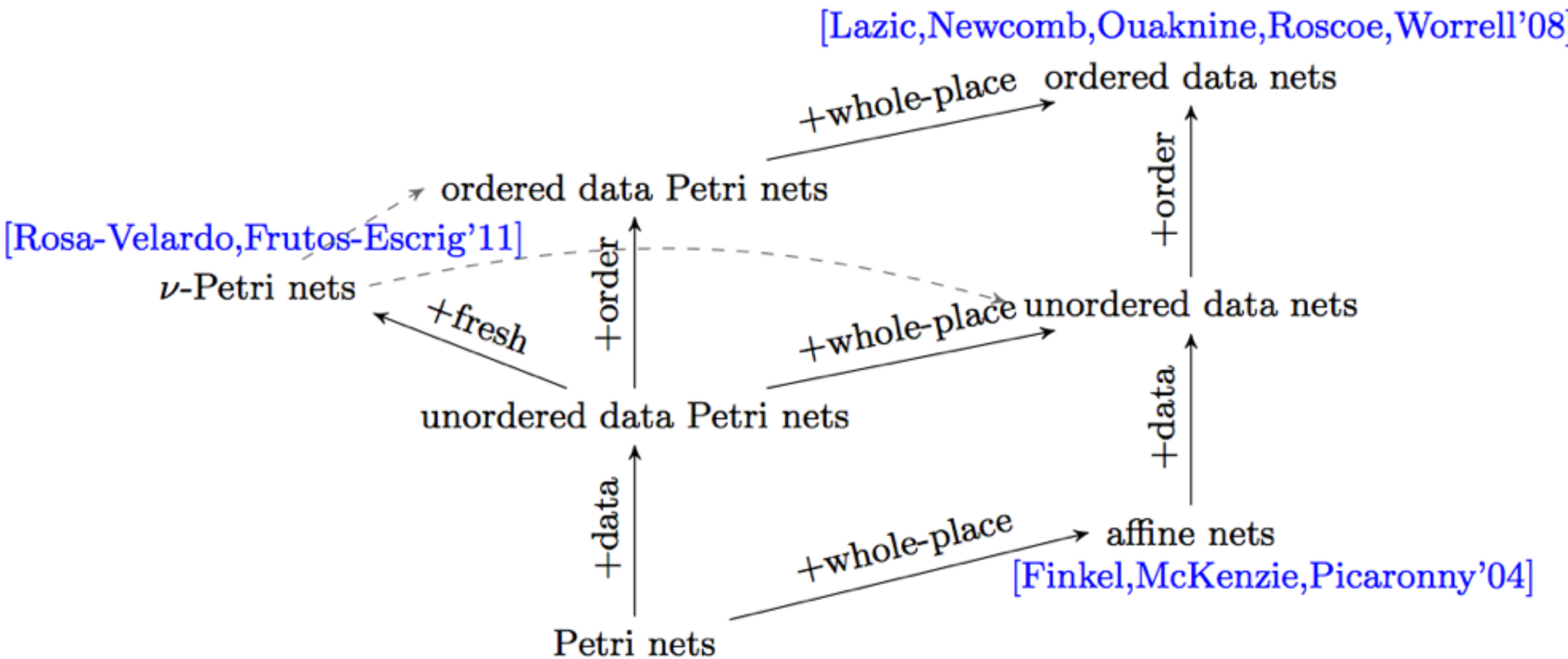
taxonomy of extensions



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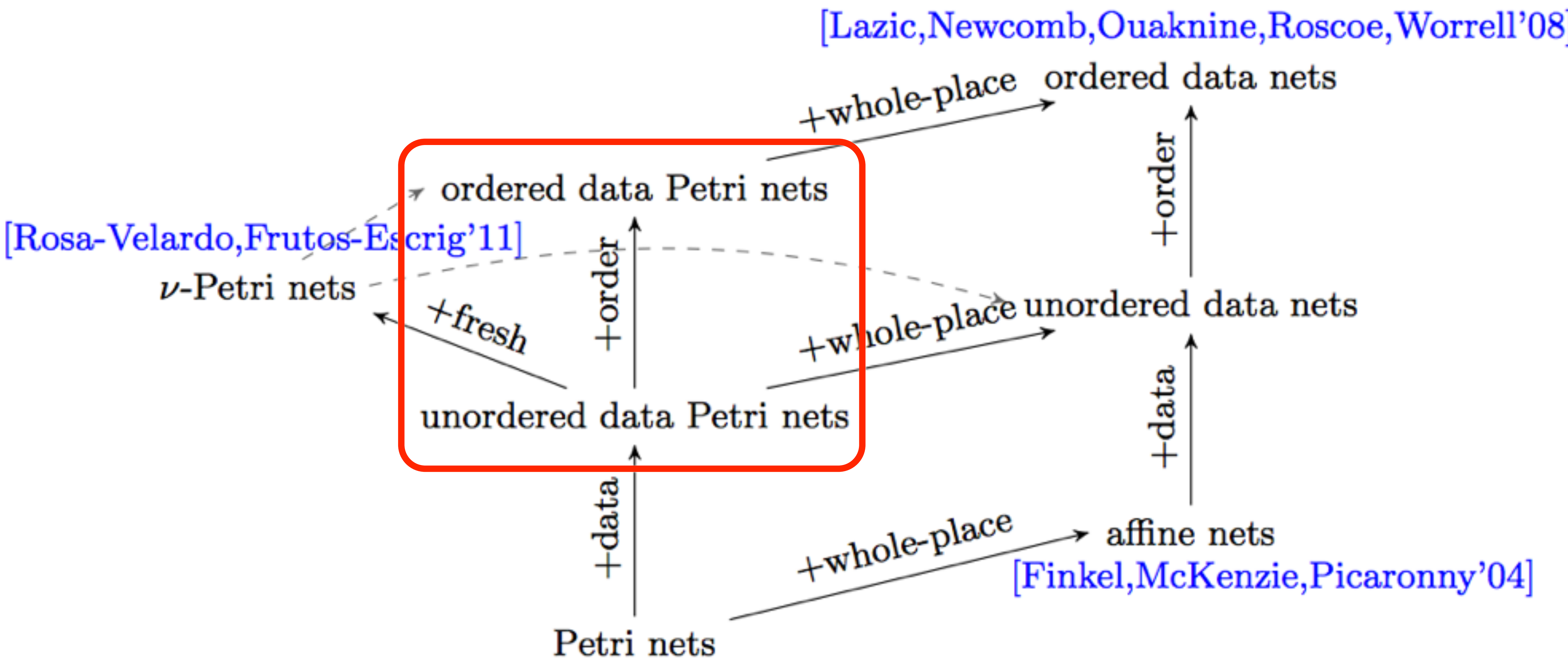

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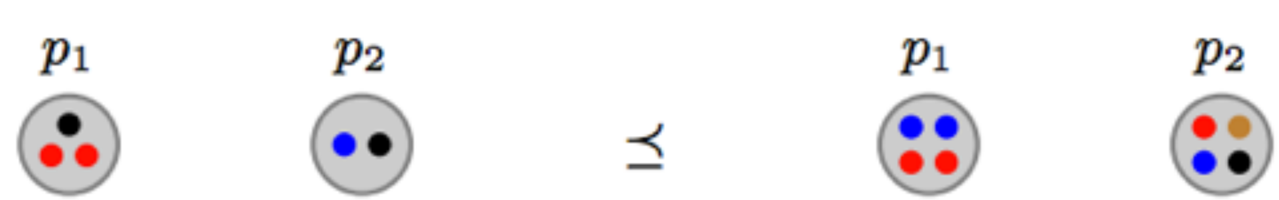

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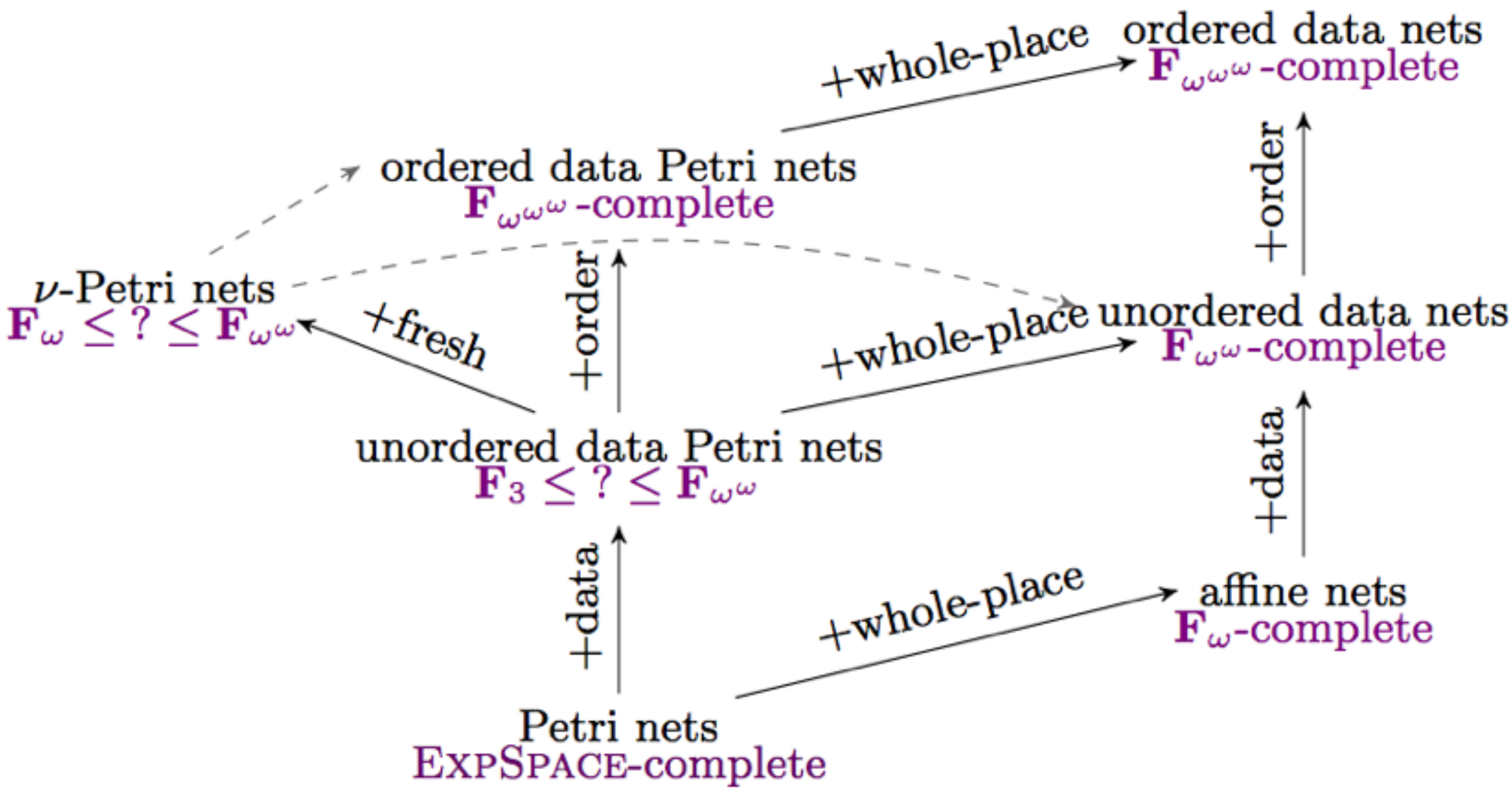

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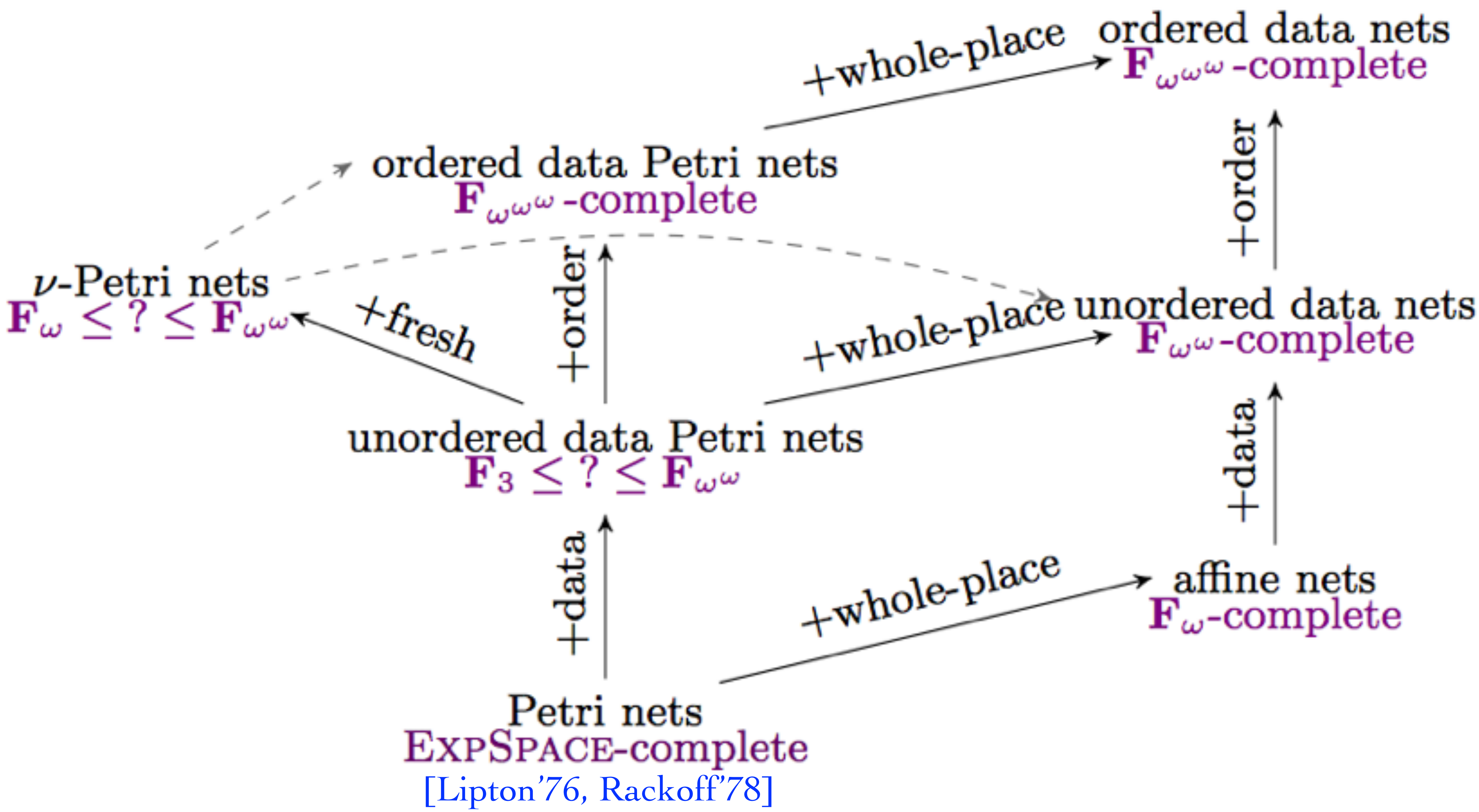

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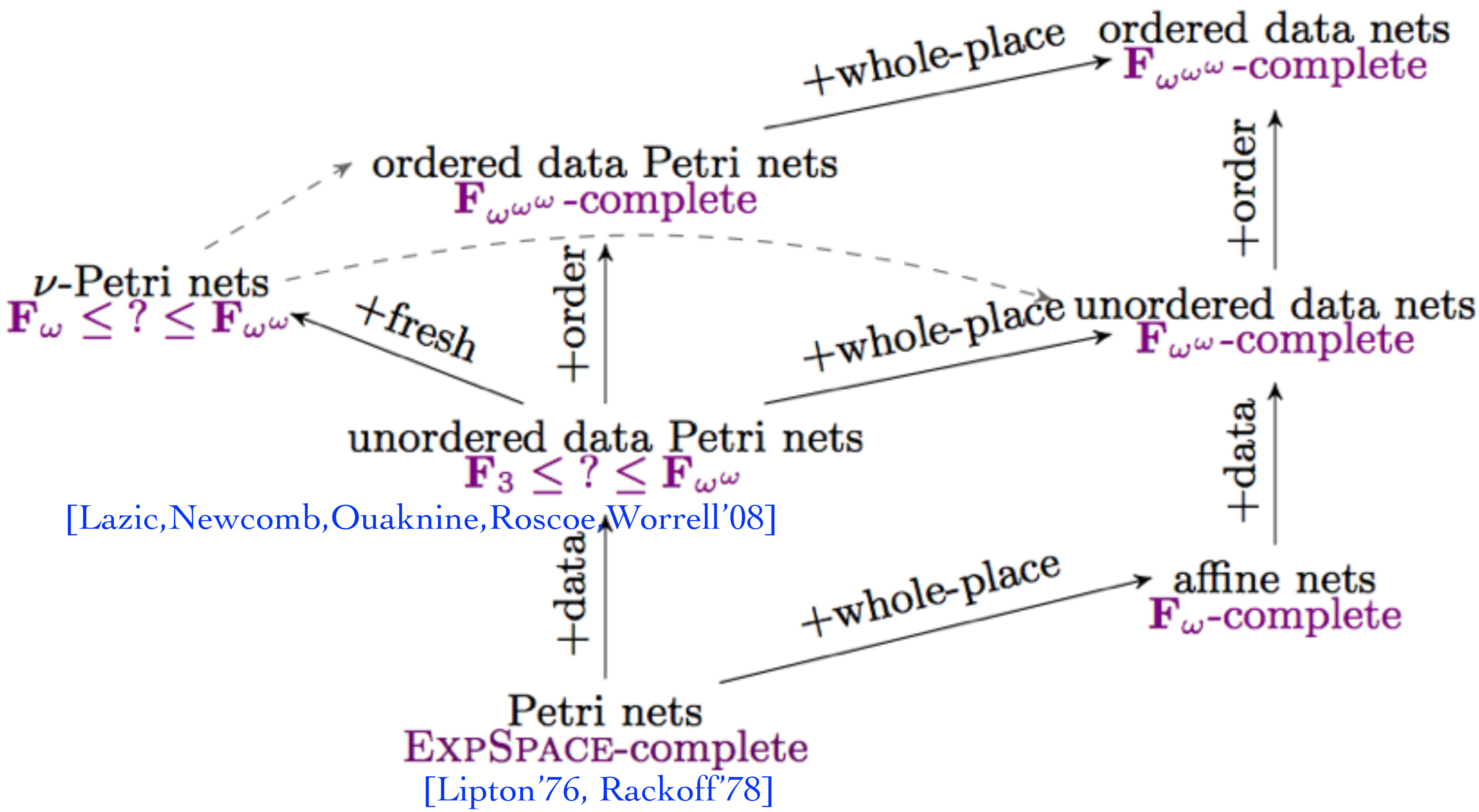

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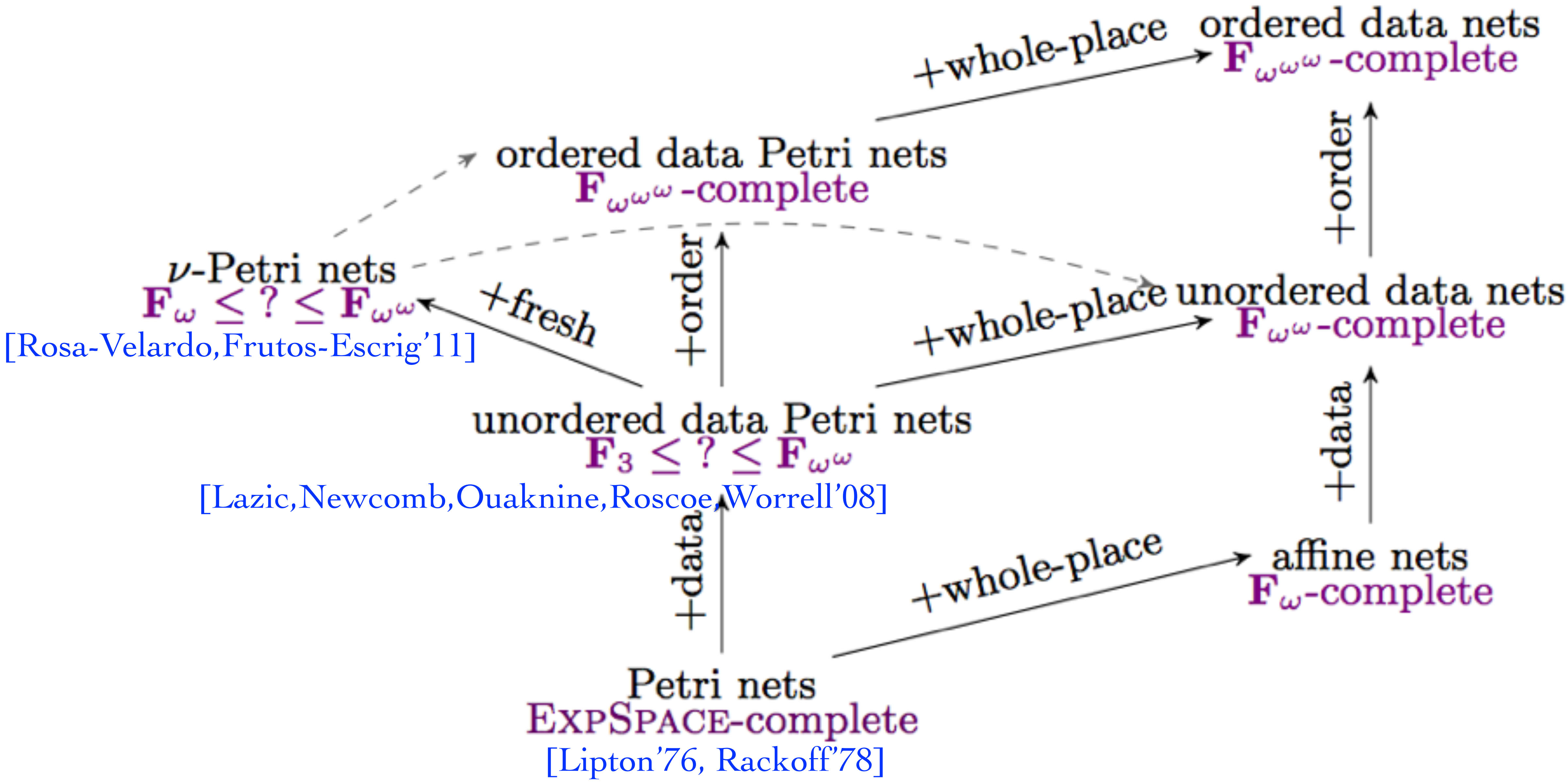

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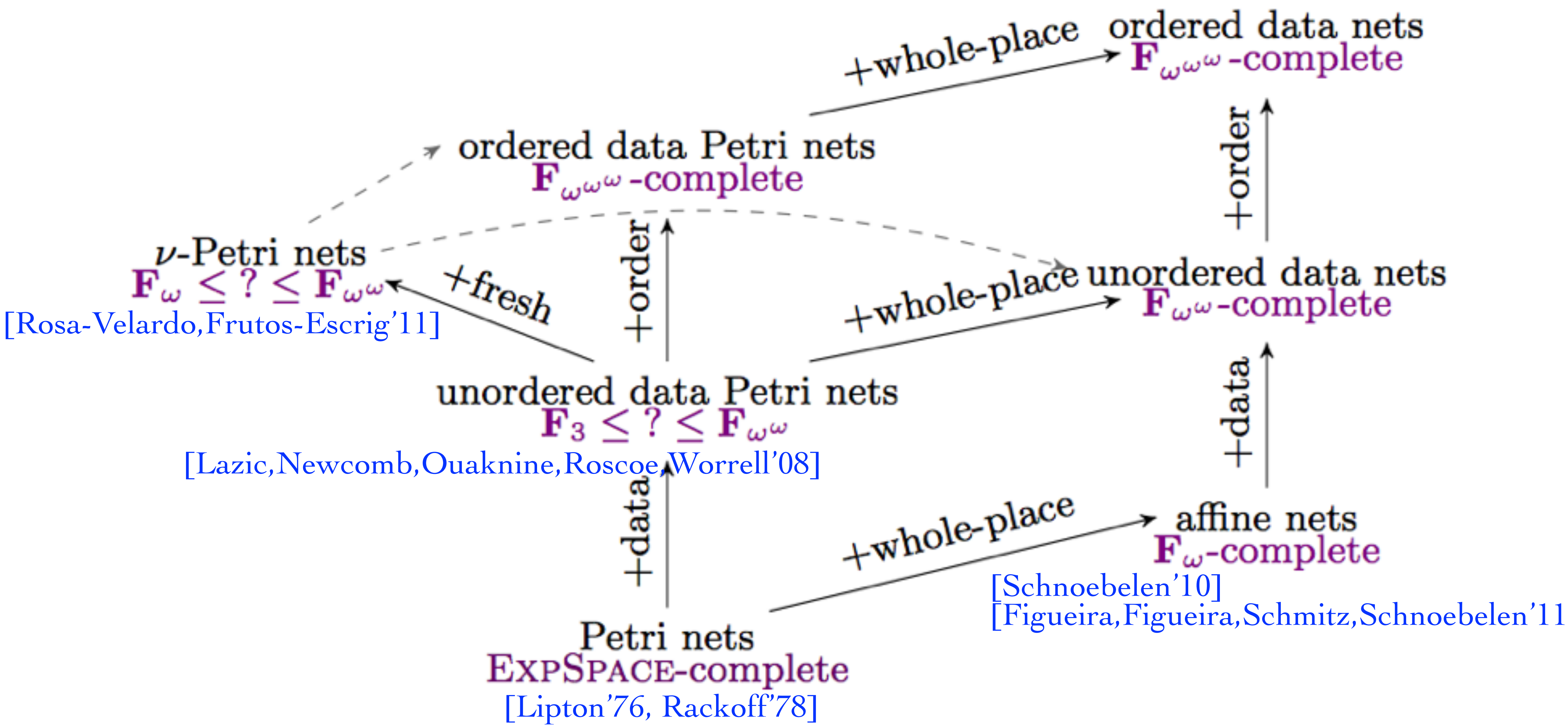

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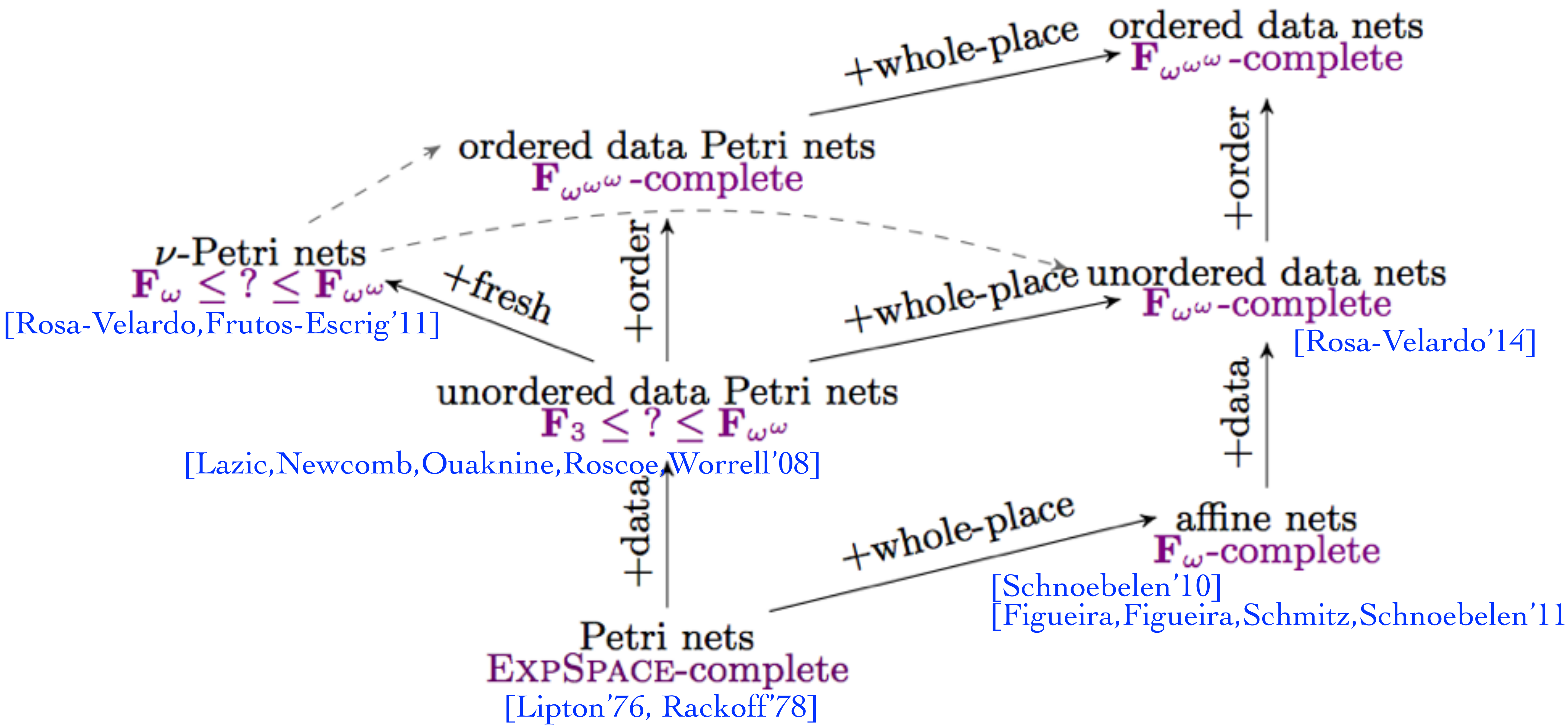

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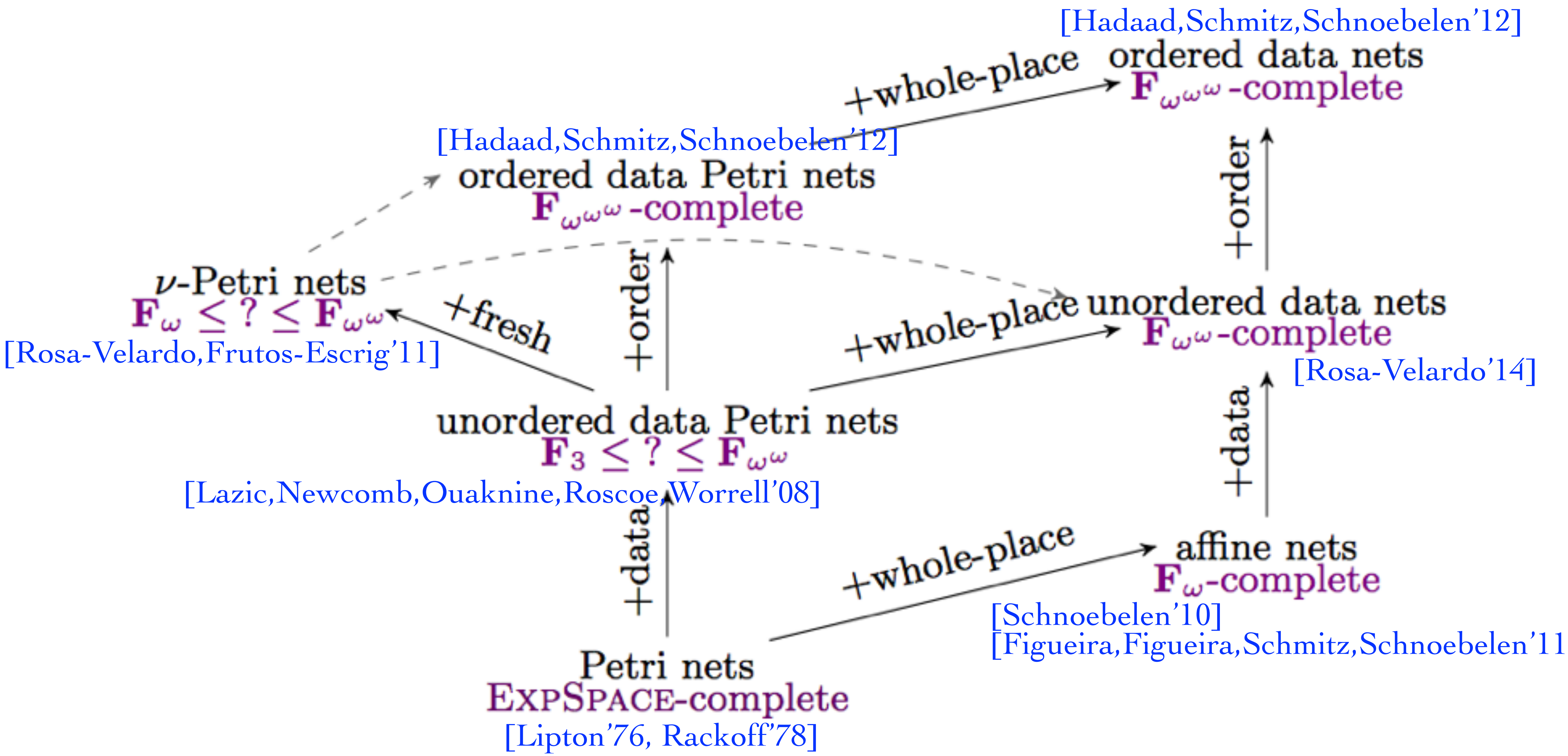

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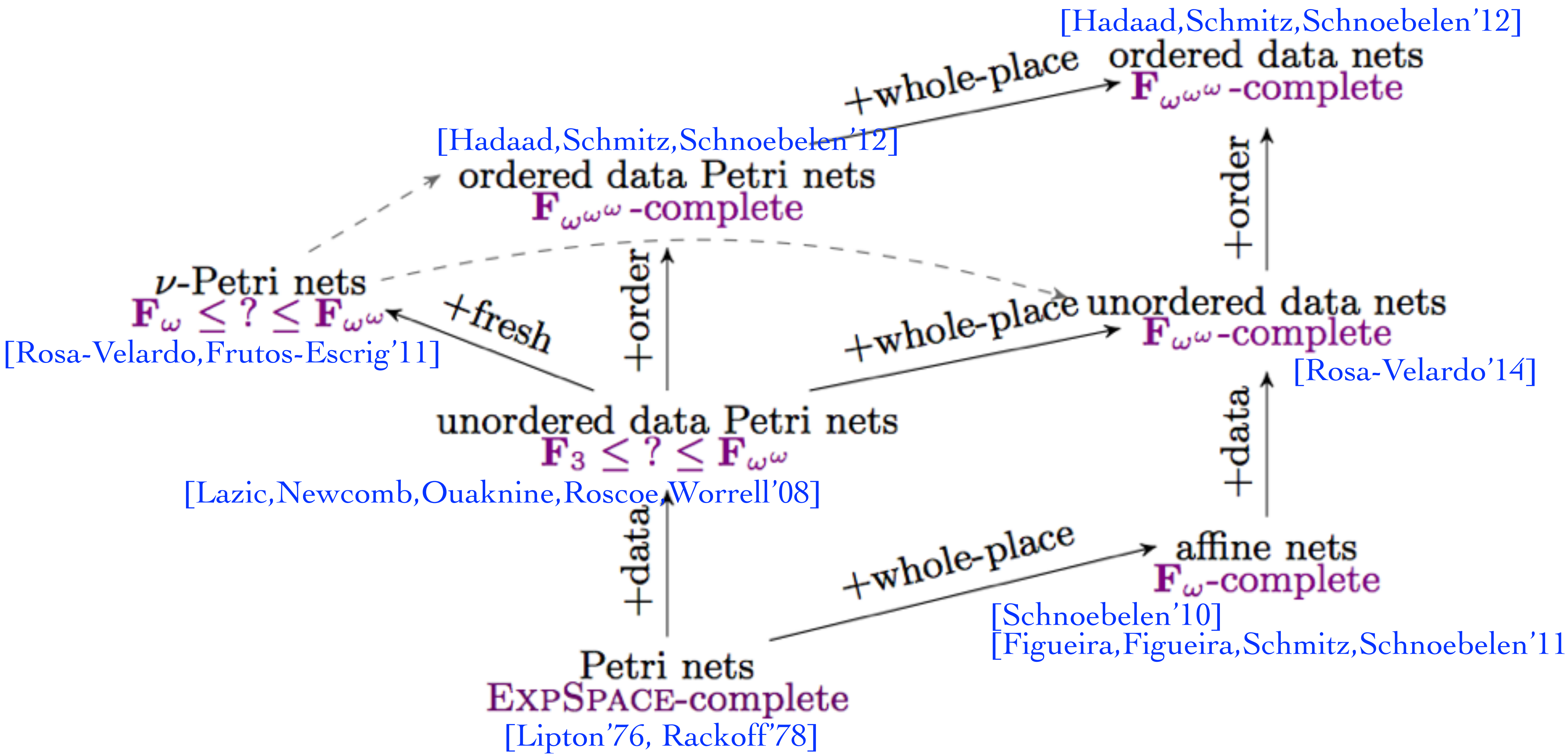

Boundedness undecidable if resets are allowed.

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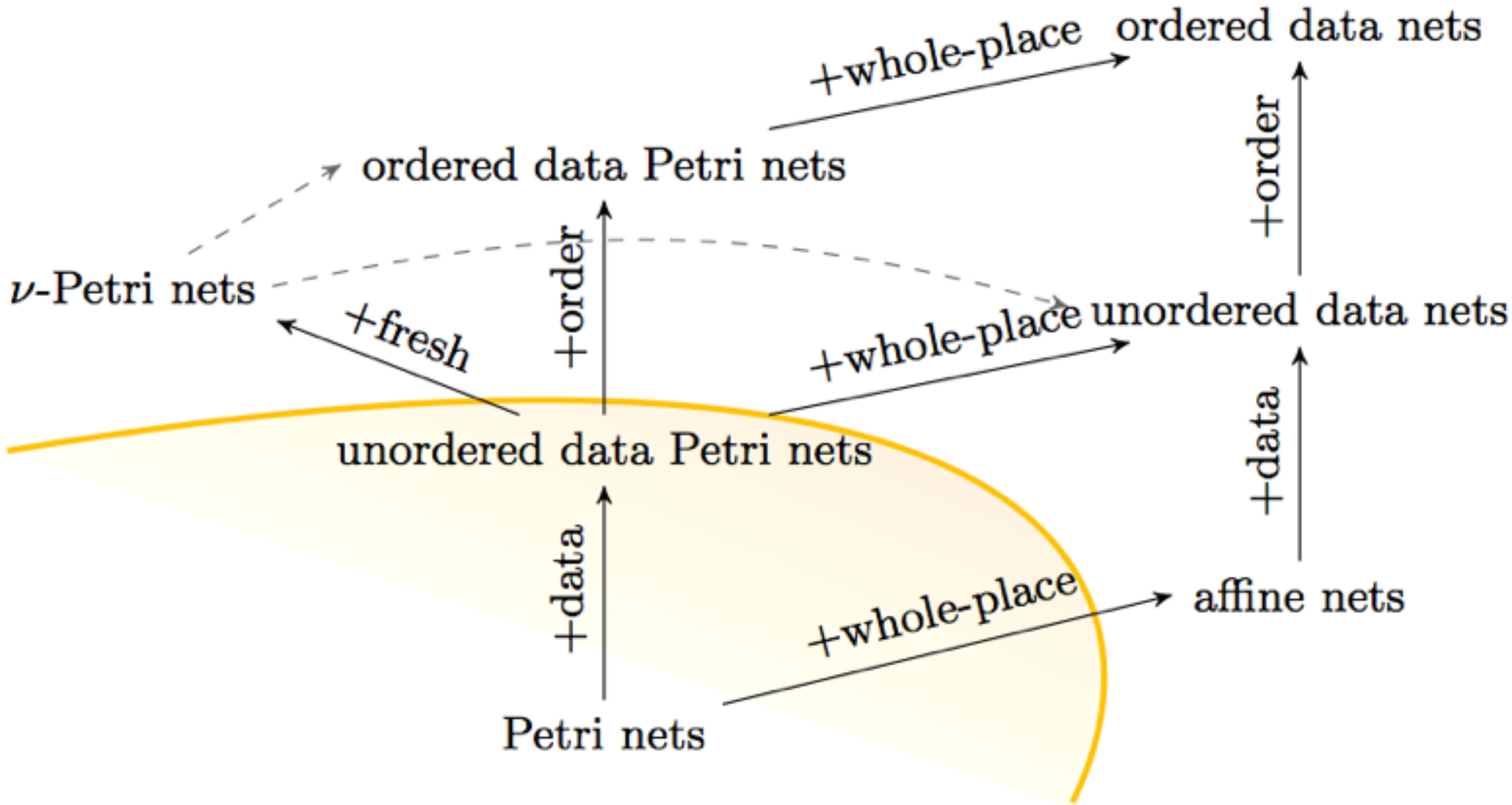
} only makes sense in data setting

decidability border for place boundedness



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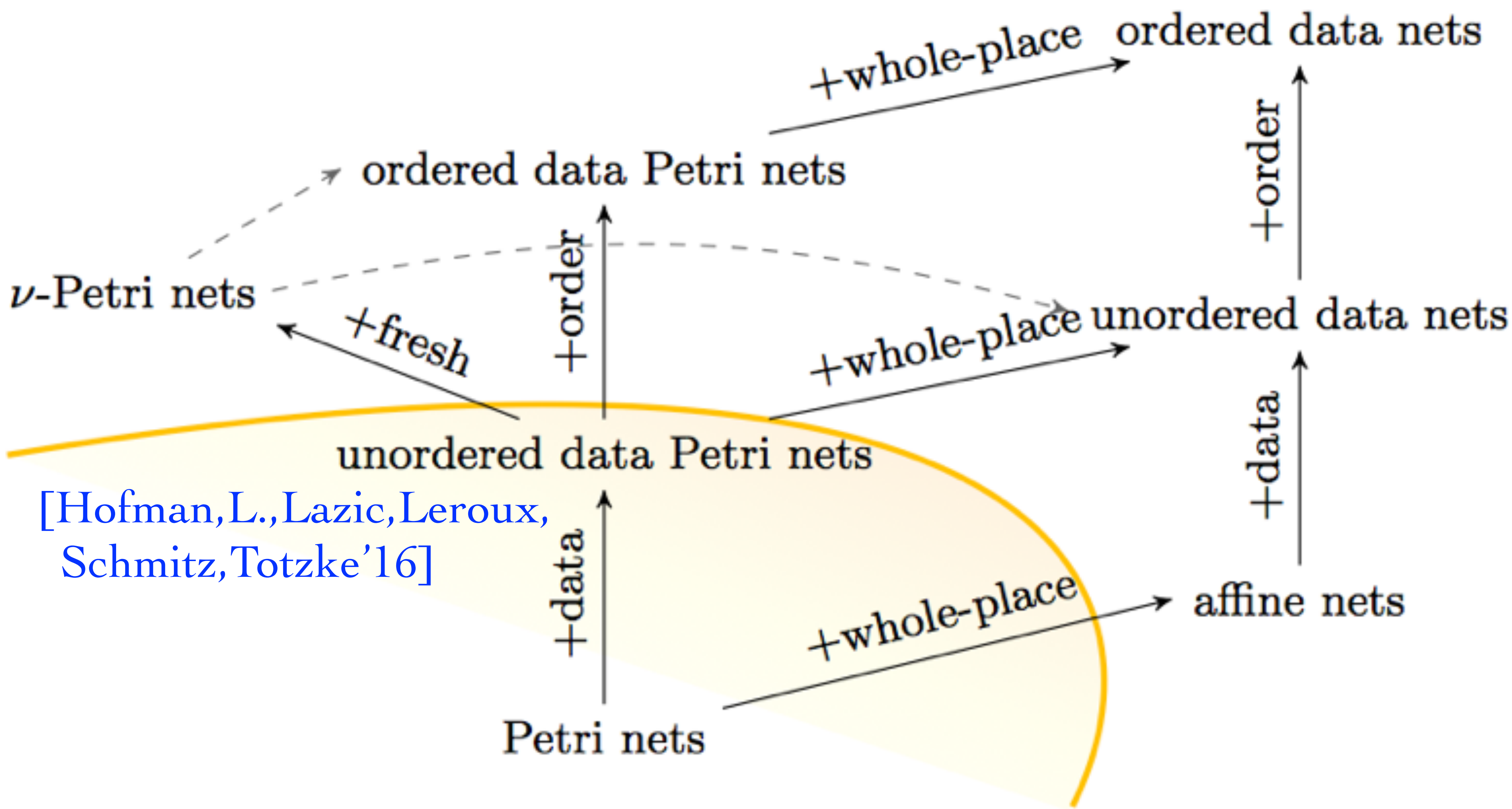

[Hofman,L.,Lazic,Leroux,
Schmitz,Totzke'16]

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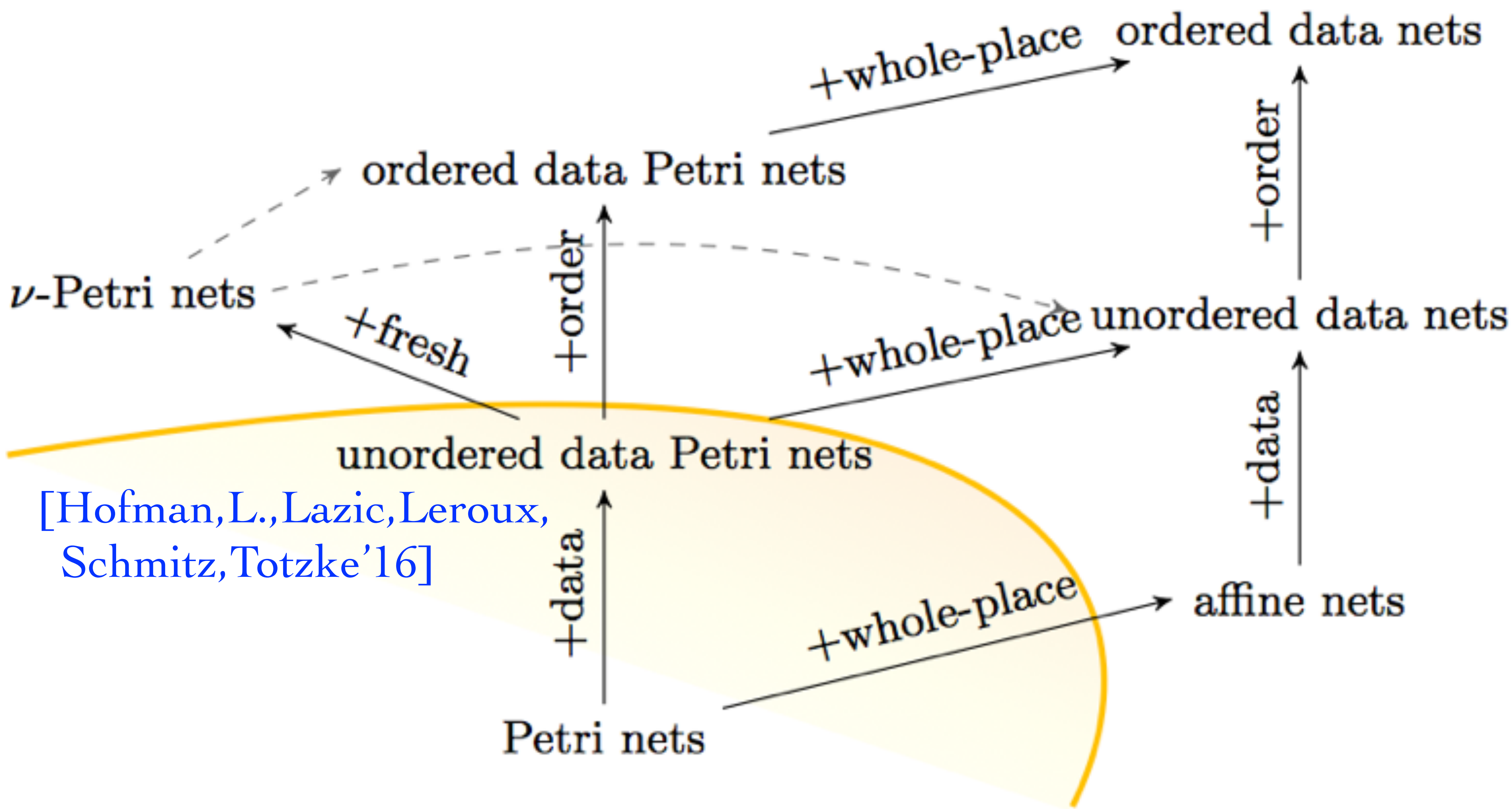

Reachability problem for unordered data Petri nets still open!

From now on we only consider standard problems like:

- termination
- coverability
- boundedness

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Fix a countably infinite relational structure $\mathbb{A}$ over a finite vocabulary, and call it data domain. This yields Petri nets with data $\mathbb{A}$.

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Data domain is a parameter in the following.

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- unordered data Petri nets = Petri nets with data $(\mathbb{N}, =)$
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- ...?

Fix a countably infinite relational structure $\mathbb{A}$ over a finite vocabulary, and call it data domain. This yields Petri nets with data $\mathbb{A}$.

Data domain is a parameter in the following.

Automorphisms of $\mathbb{A}$ we call data automorphisms.

example data domains

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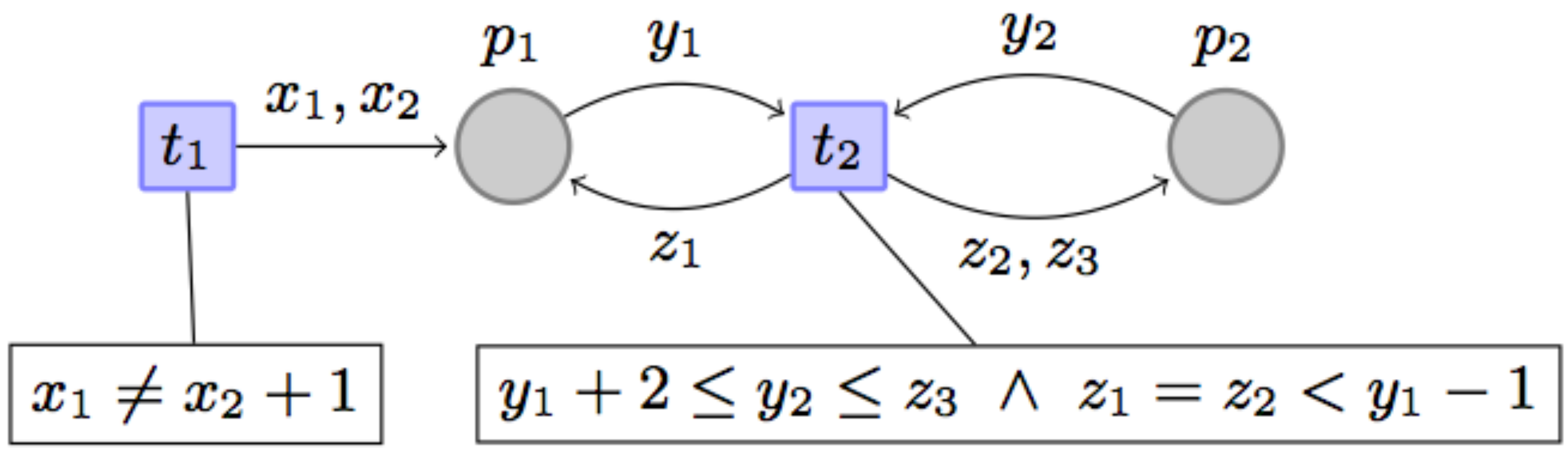

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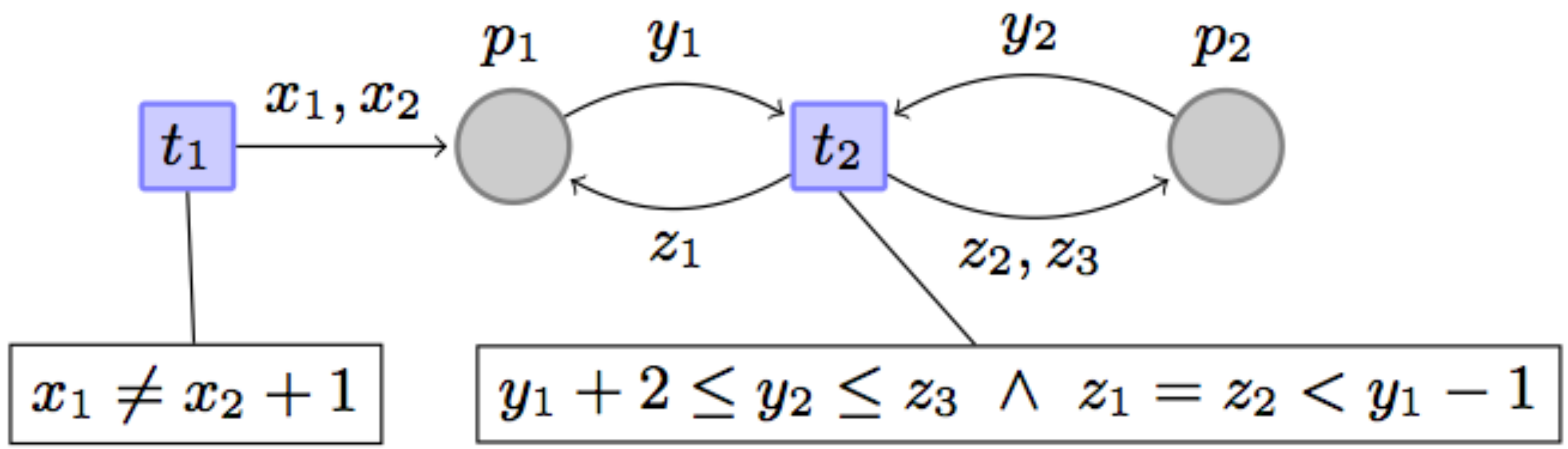

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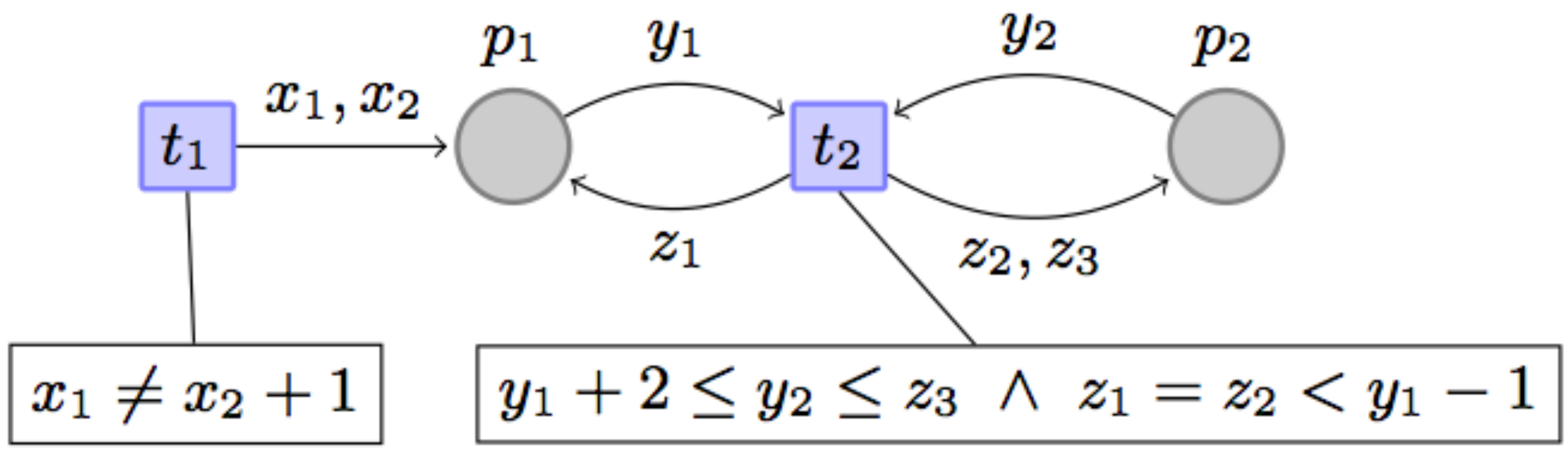

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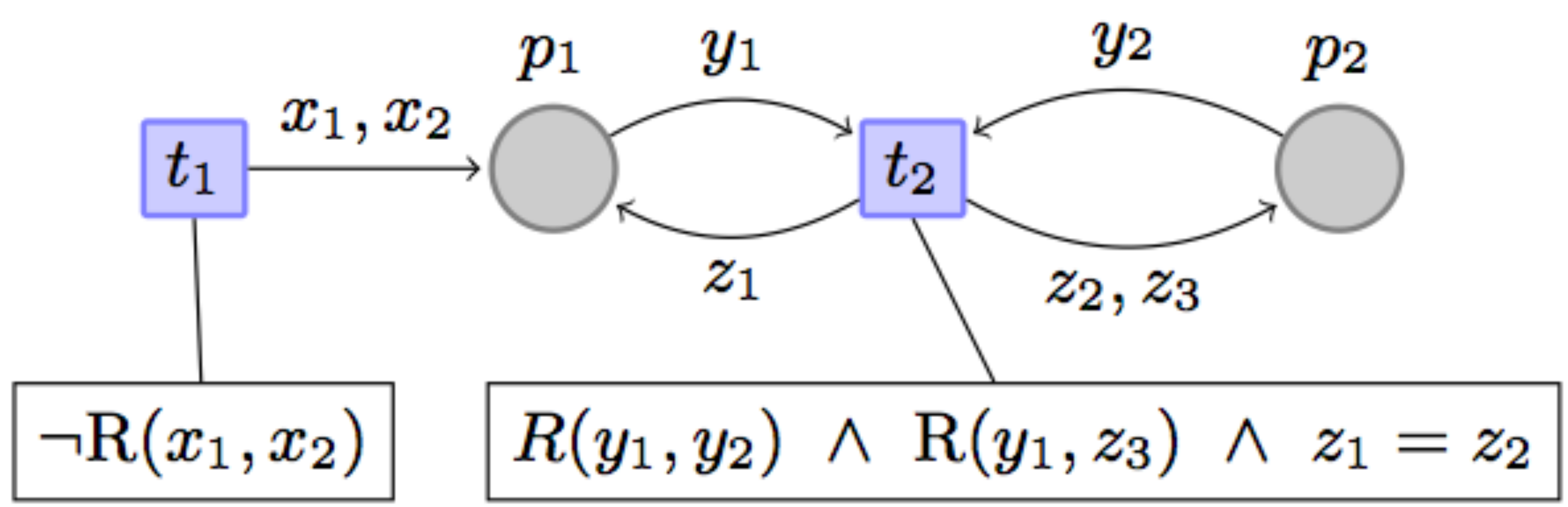

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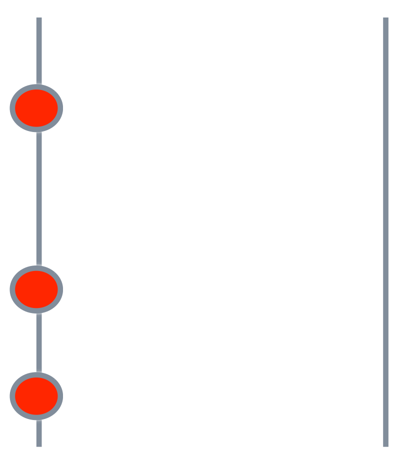

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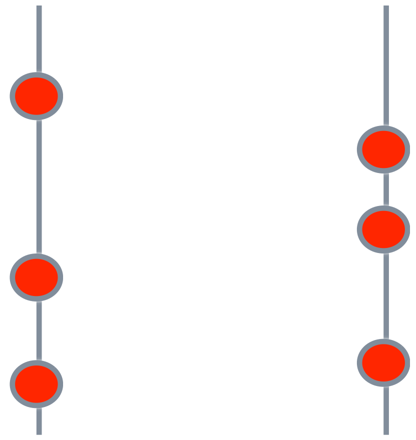

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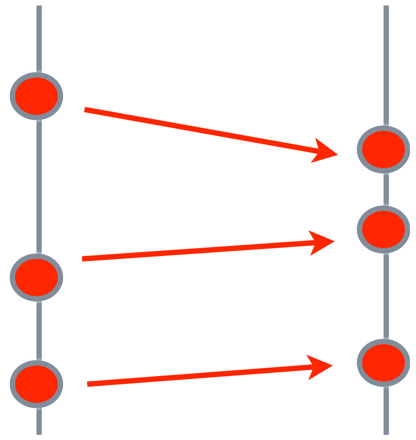

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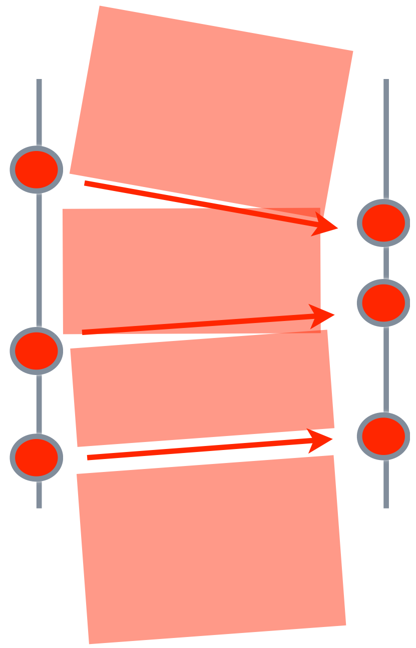

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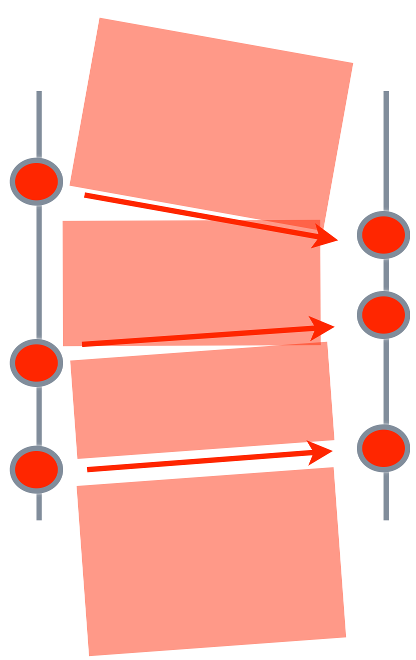

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Isomorphic configurations are equal up to data automorphism.

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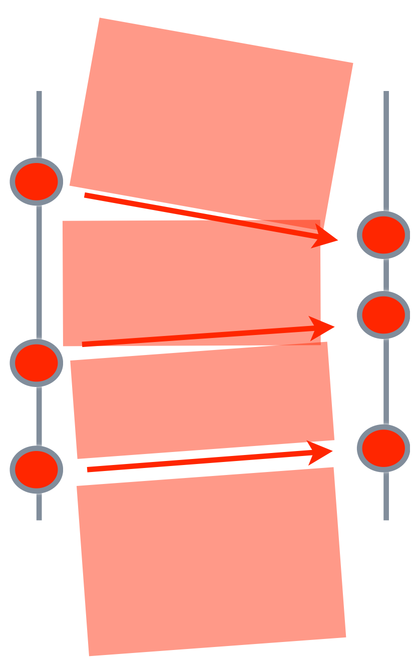

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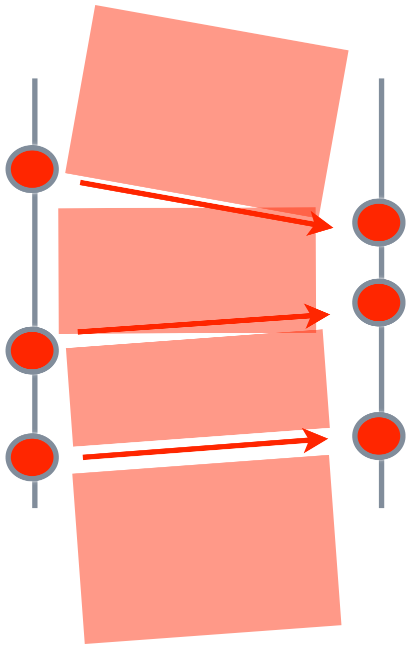
Thus a configuration can be finitely represented by its isomorphism type.

homogeneous data domains



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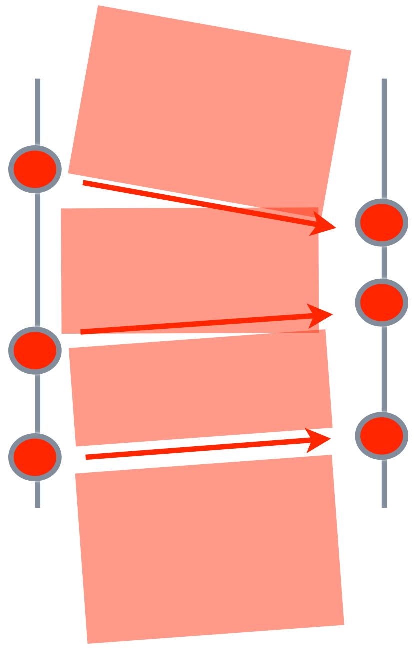

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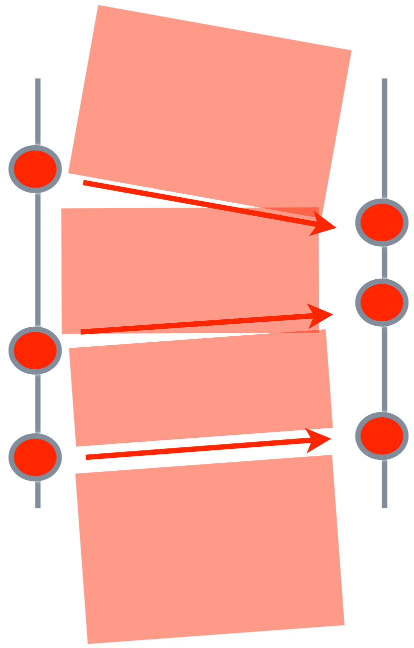

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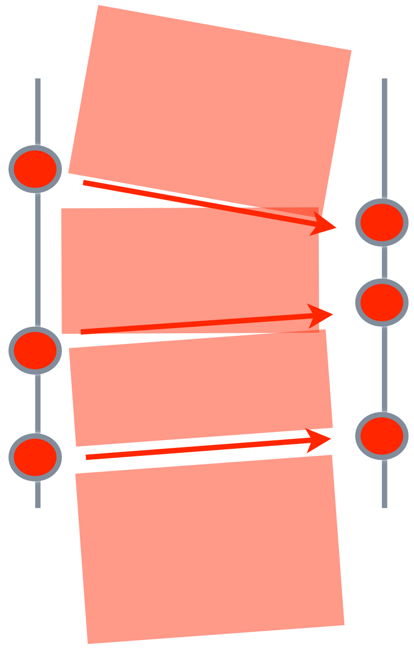

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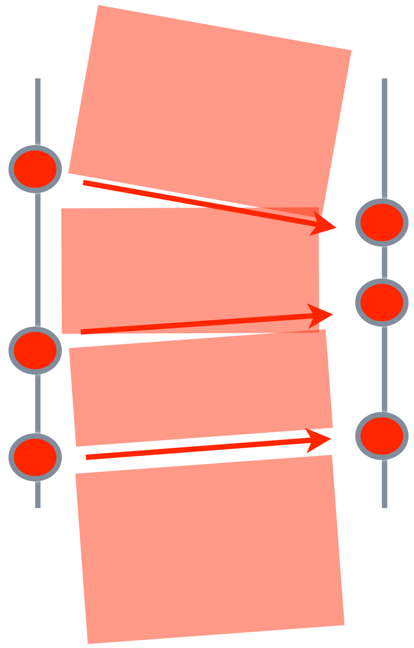

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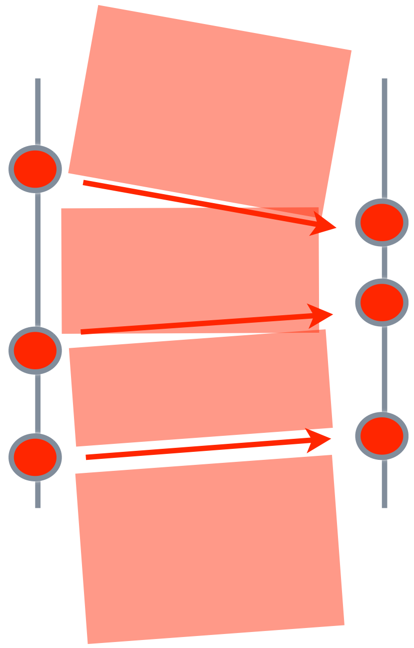

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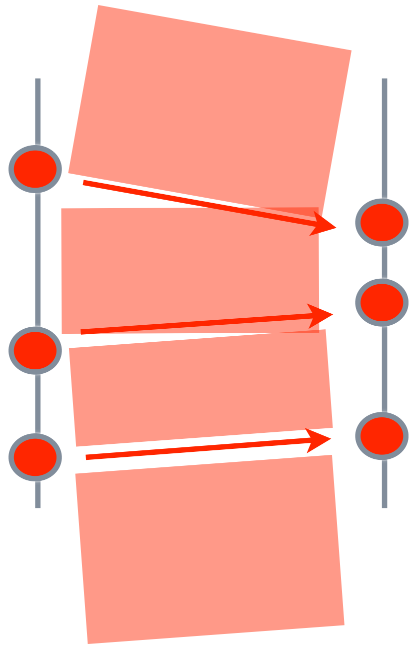

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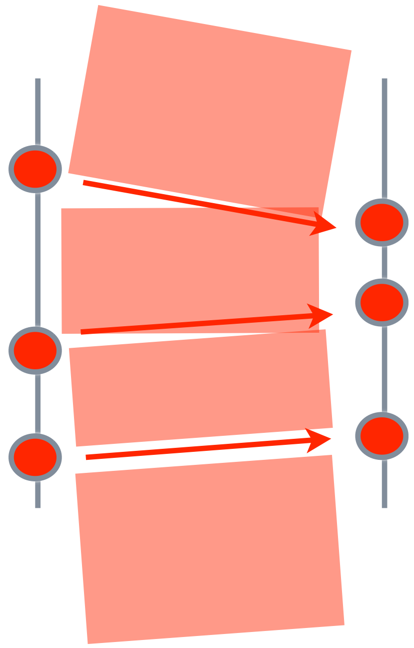

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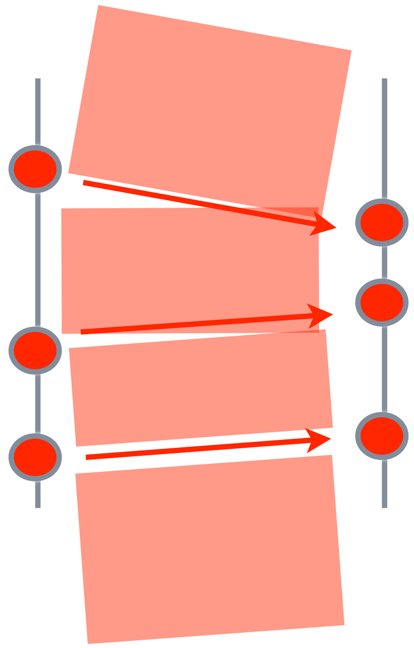

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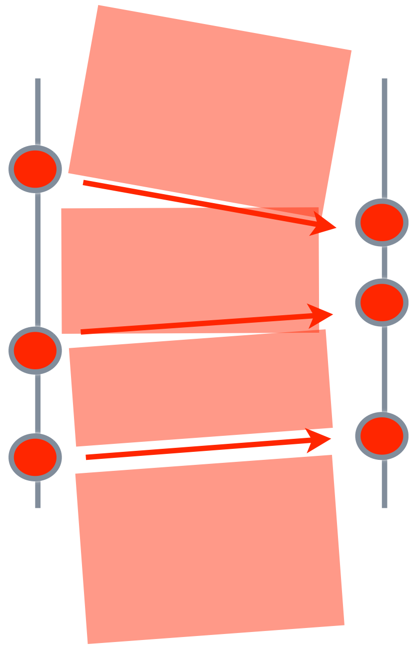

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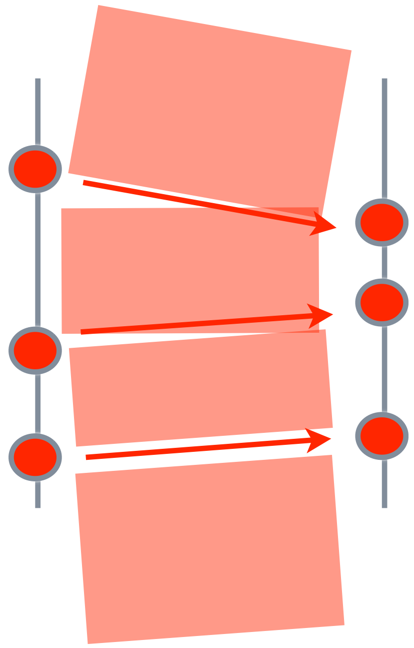

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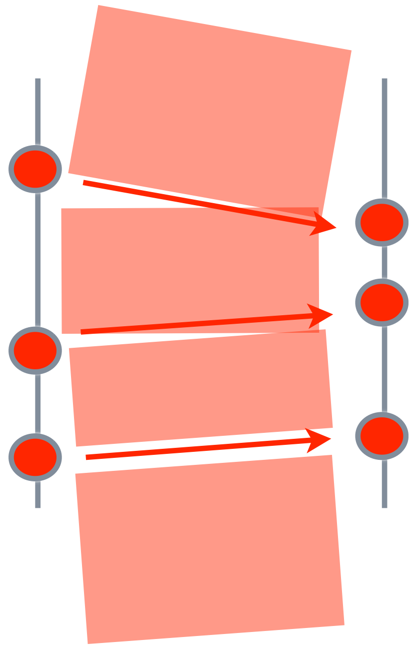

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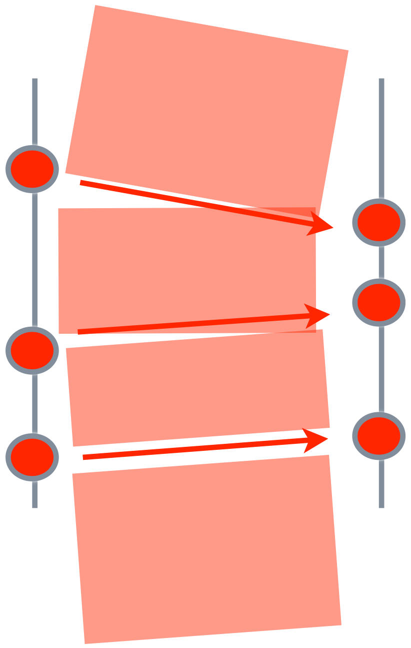
quantifier elimination



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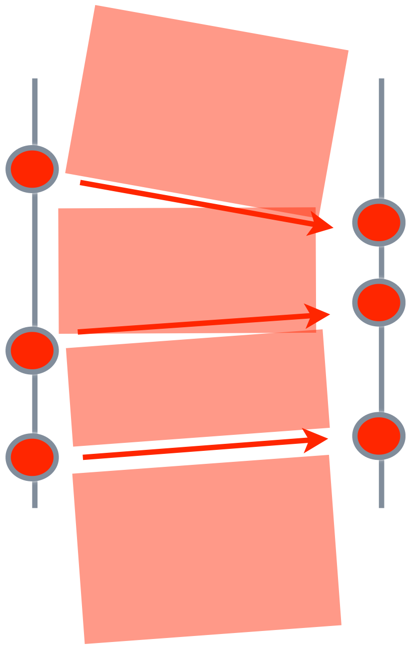

Theorem:

Homogeneous structures admit quantifier elimination: every first-order formula is equivalent to a quantifier-free one.

amalgamation classes

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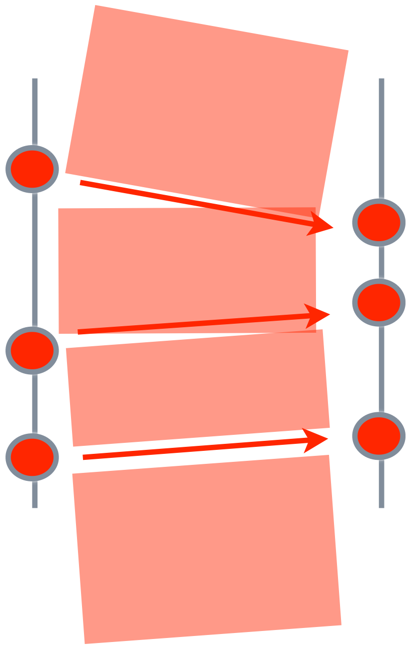

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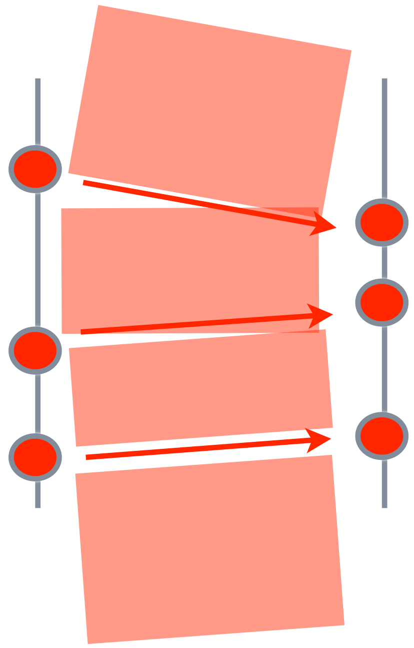

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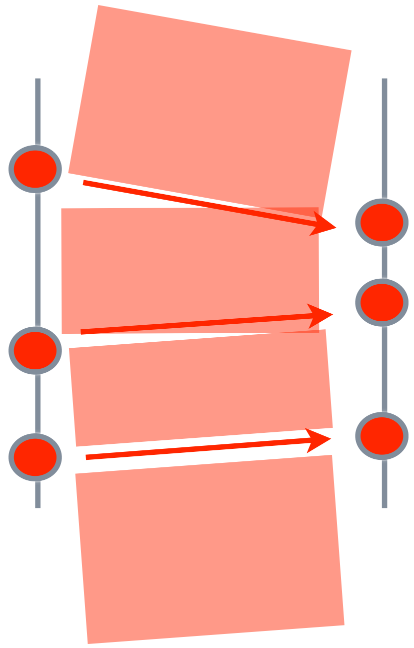

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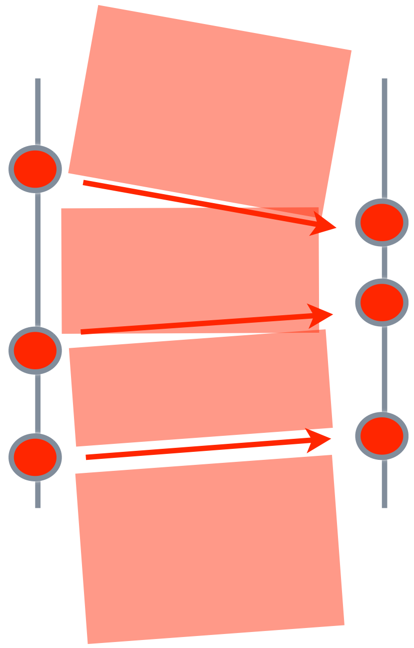

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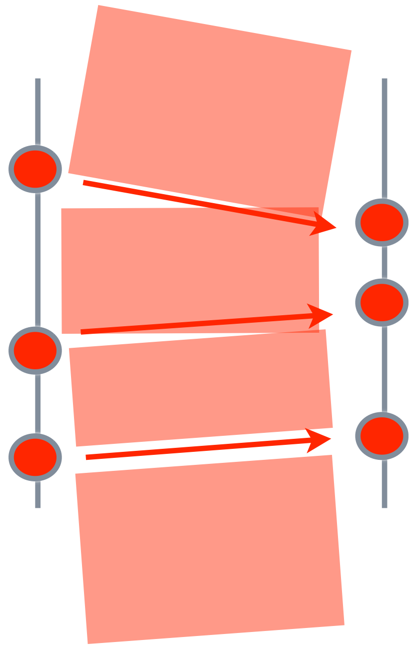

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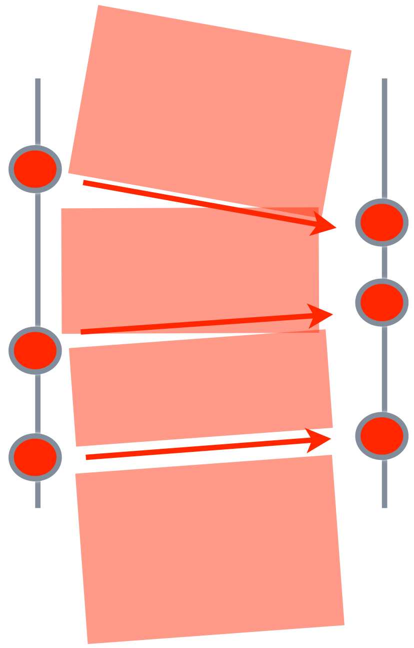

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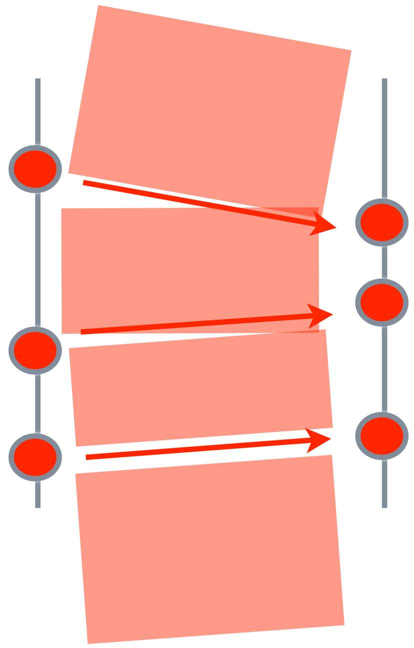

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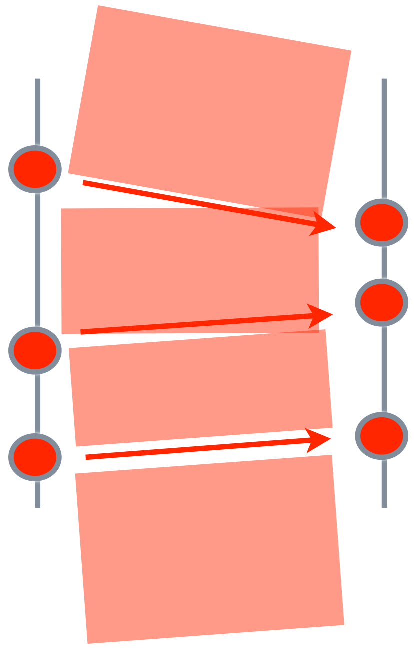

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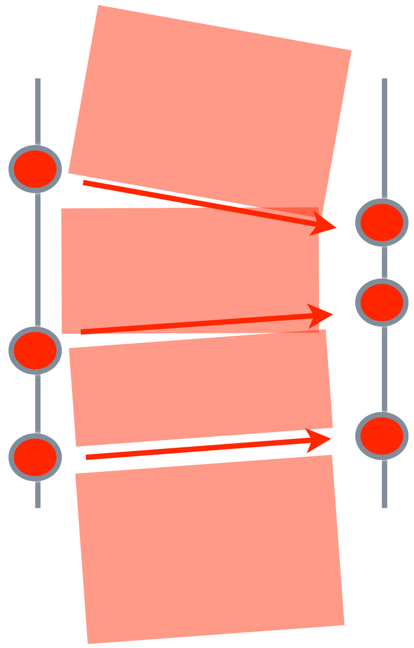

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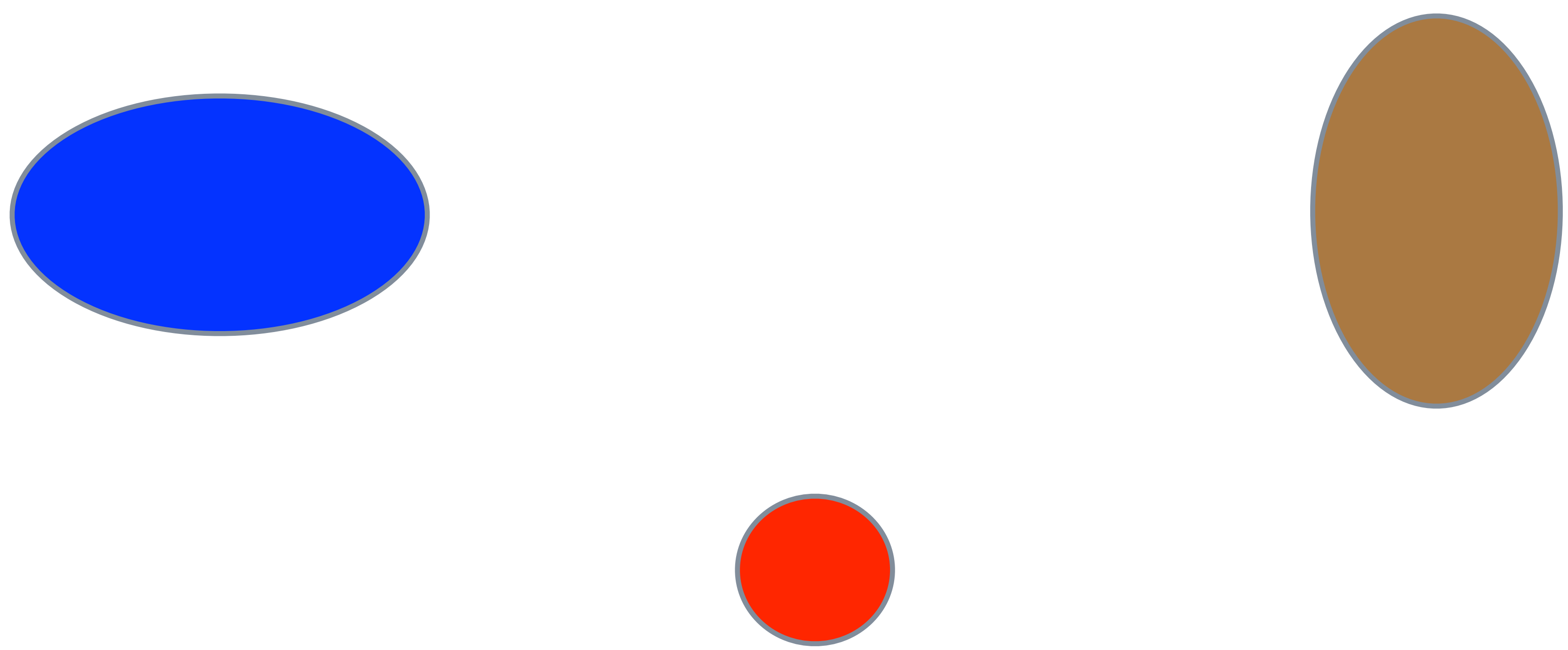

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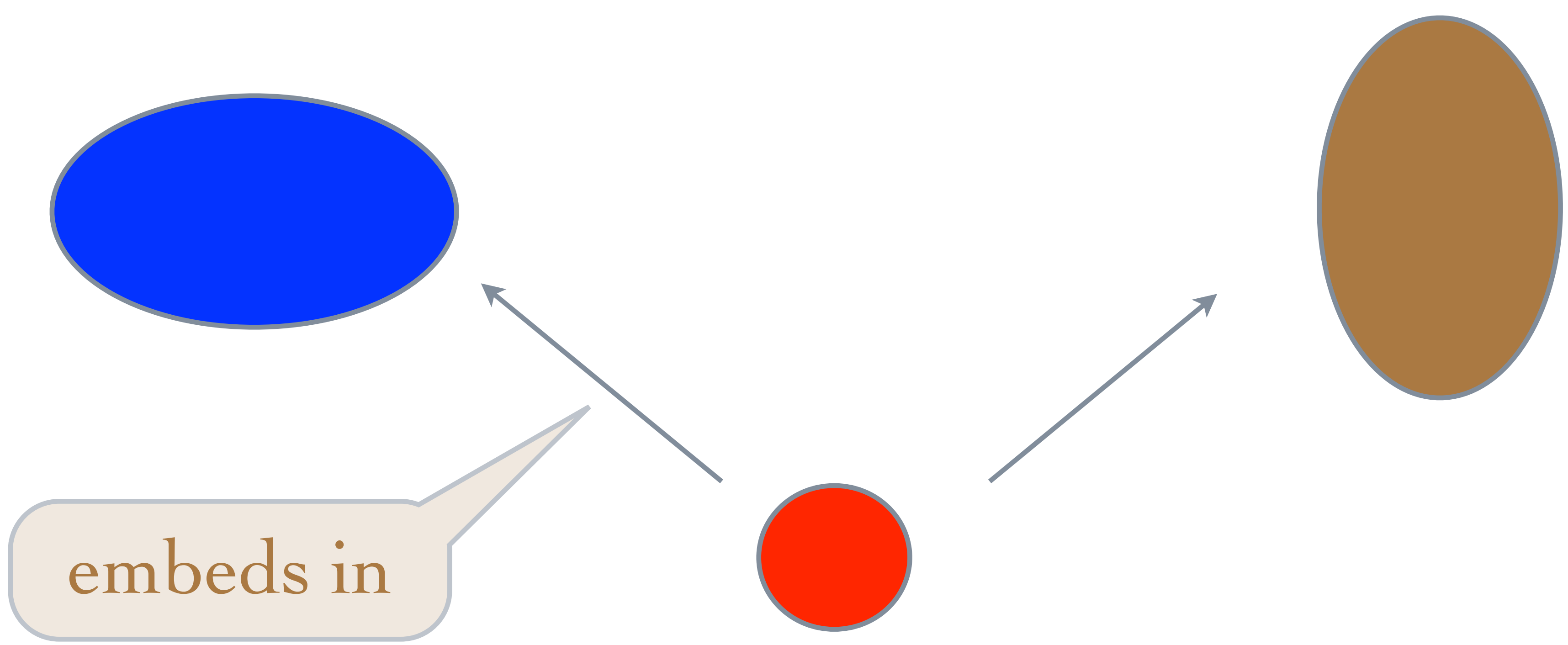

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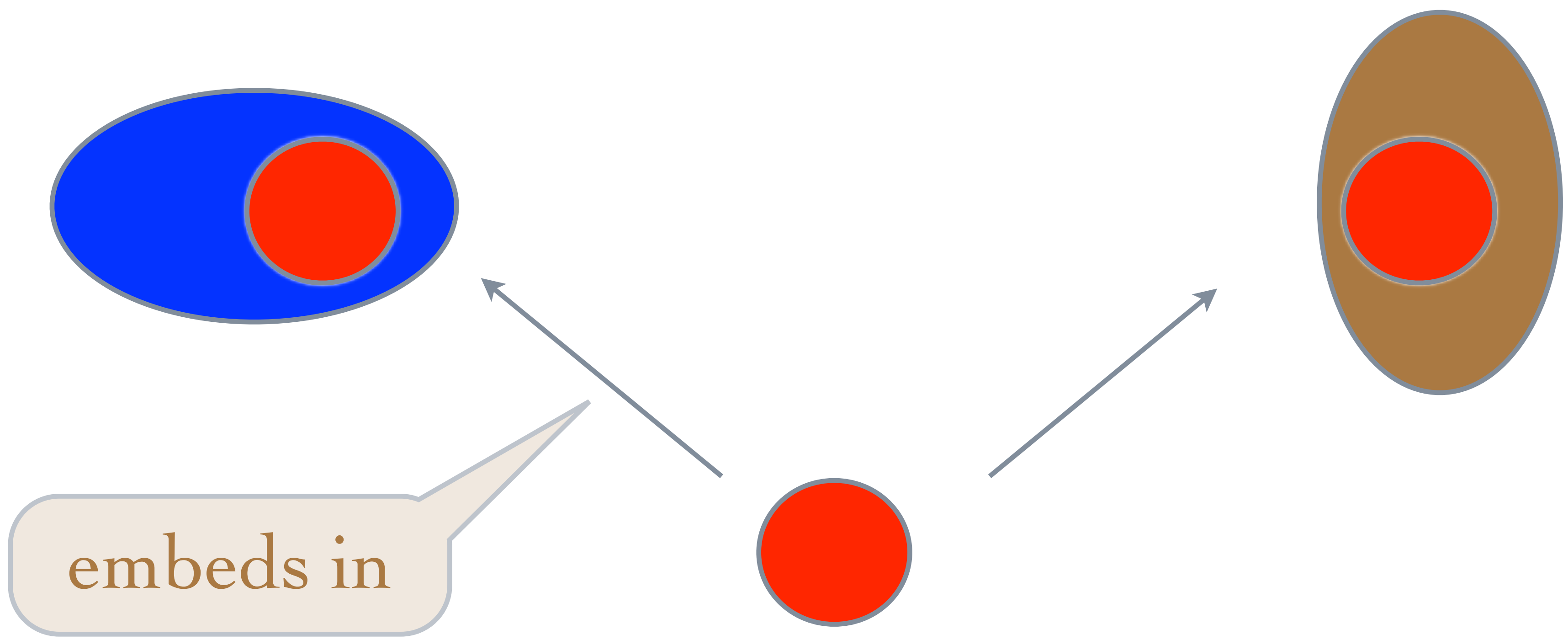

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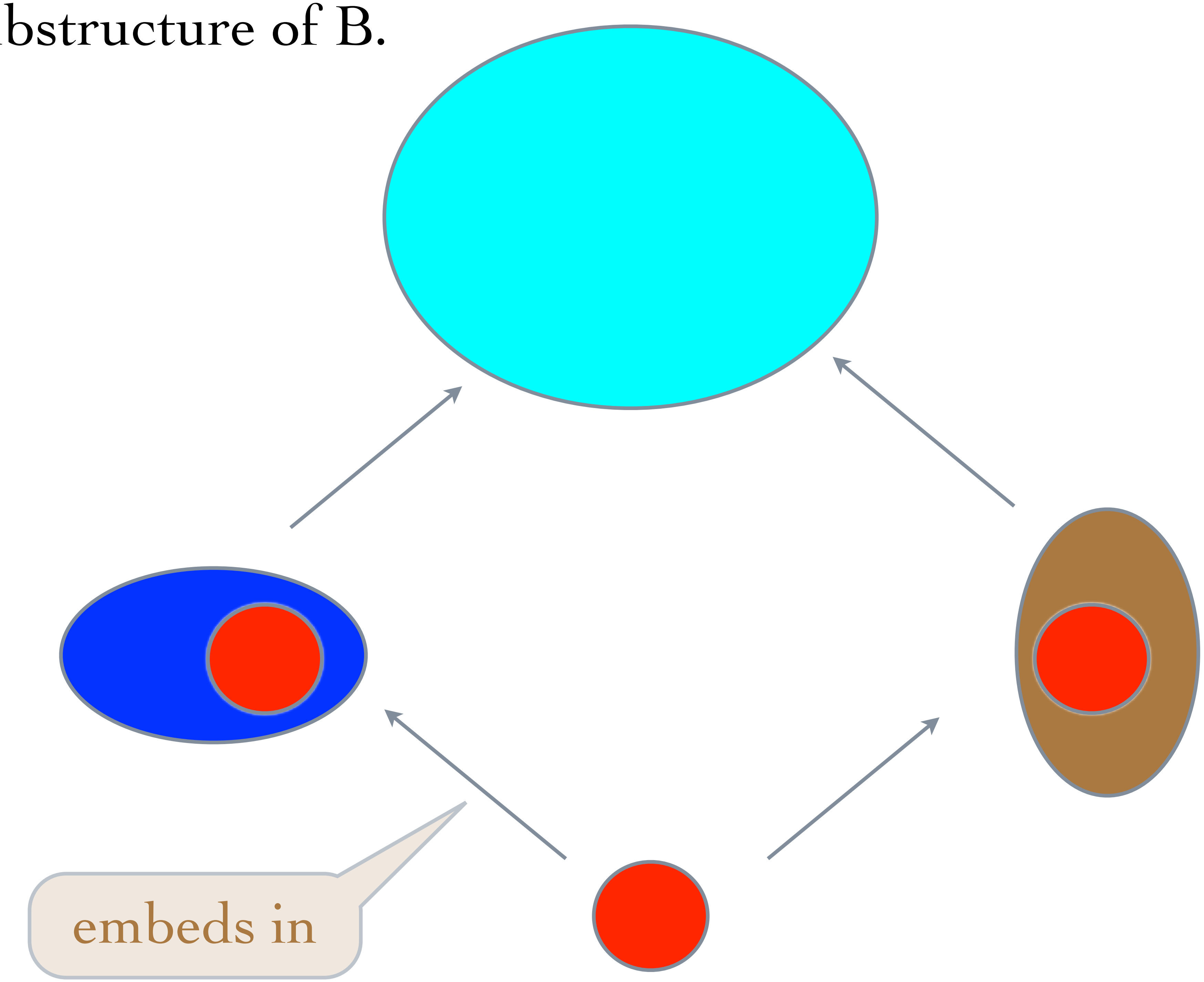

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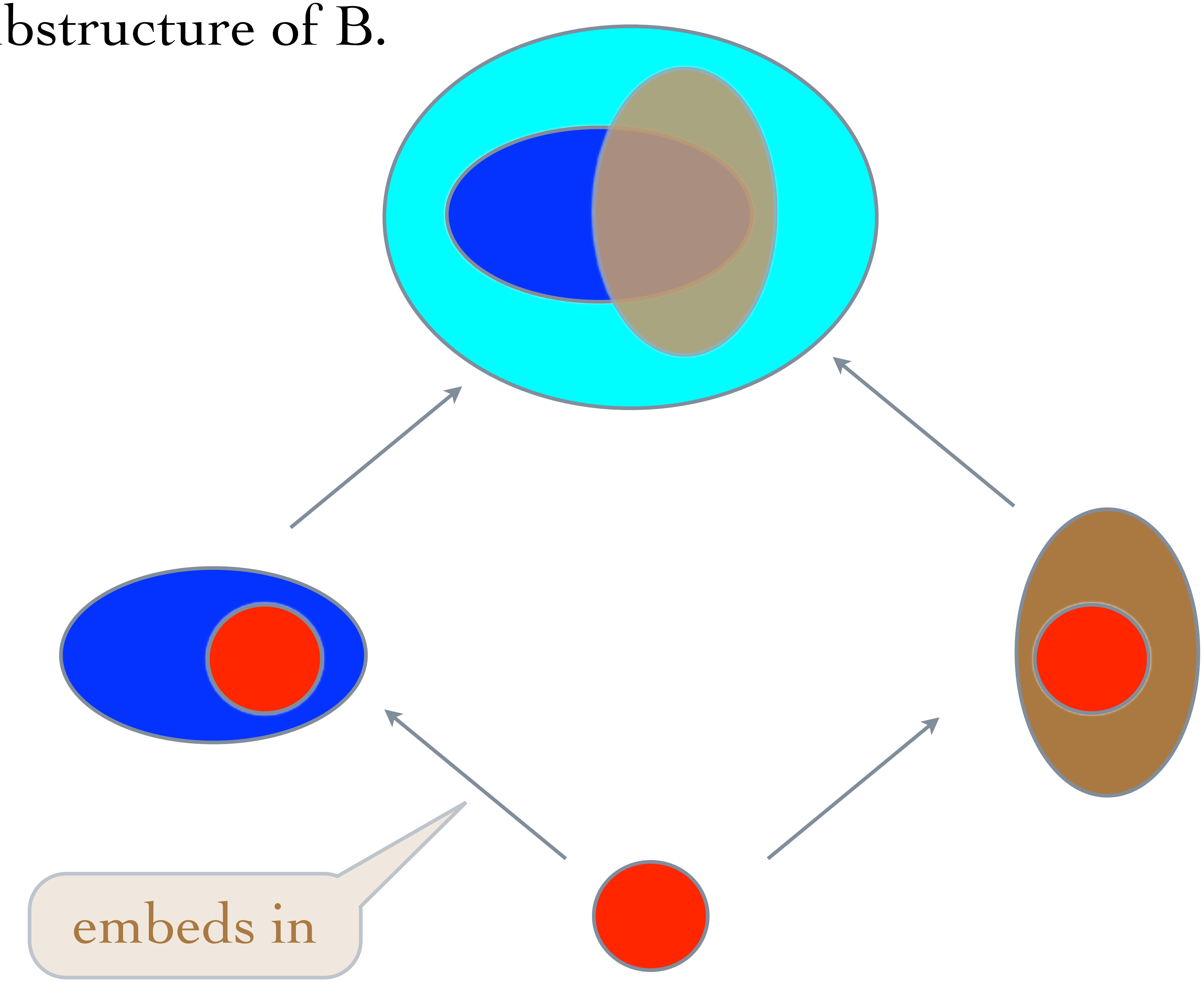

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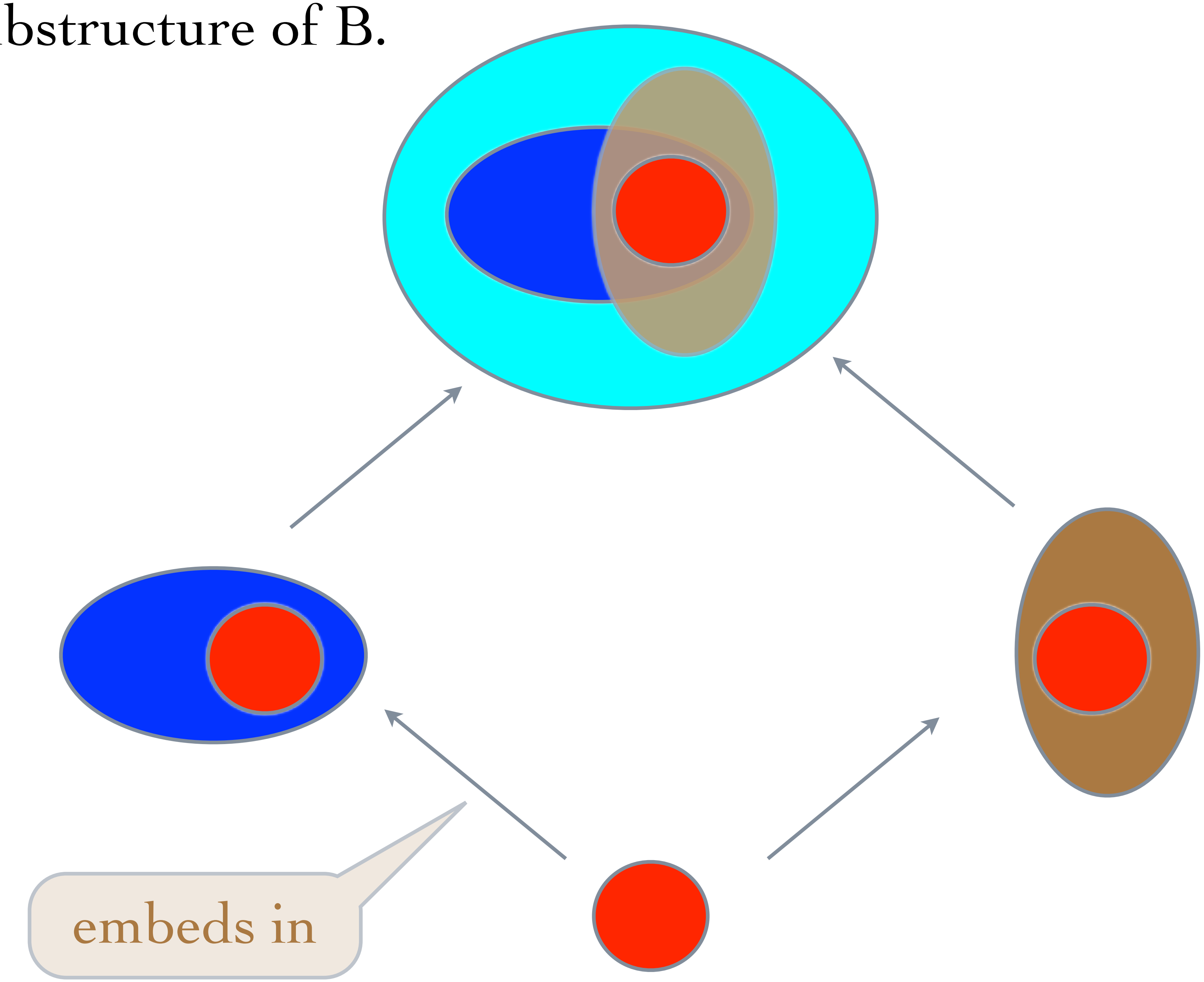

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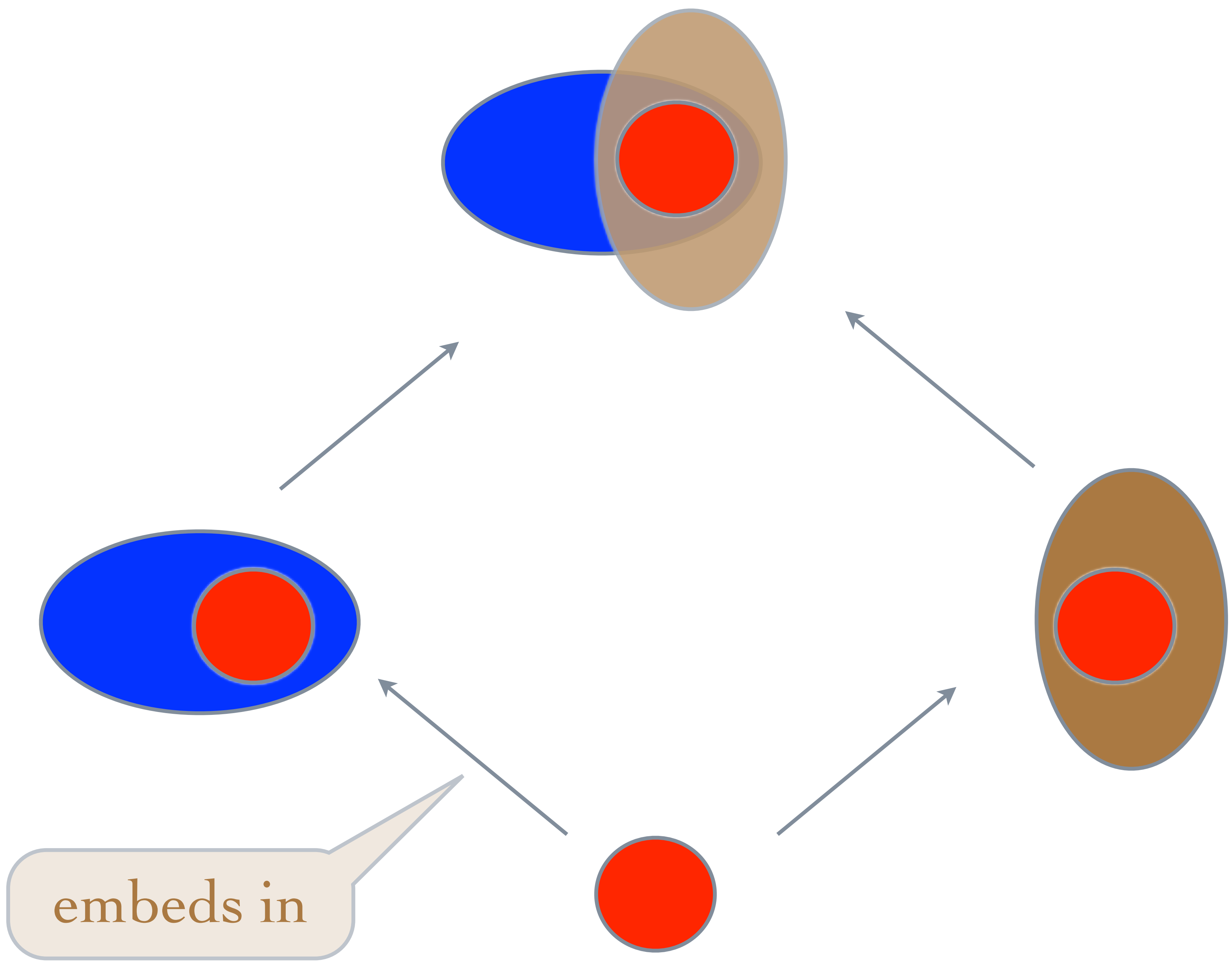

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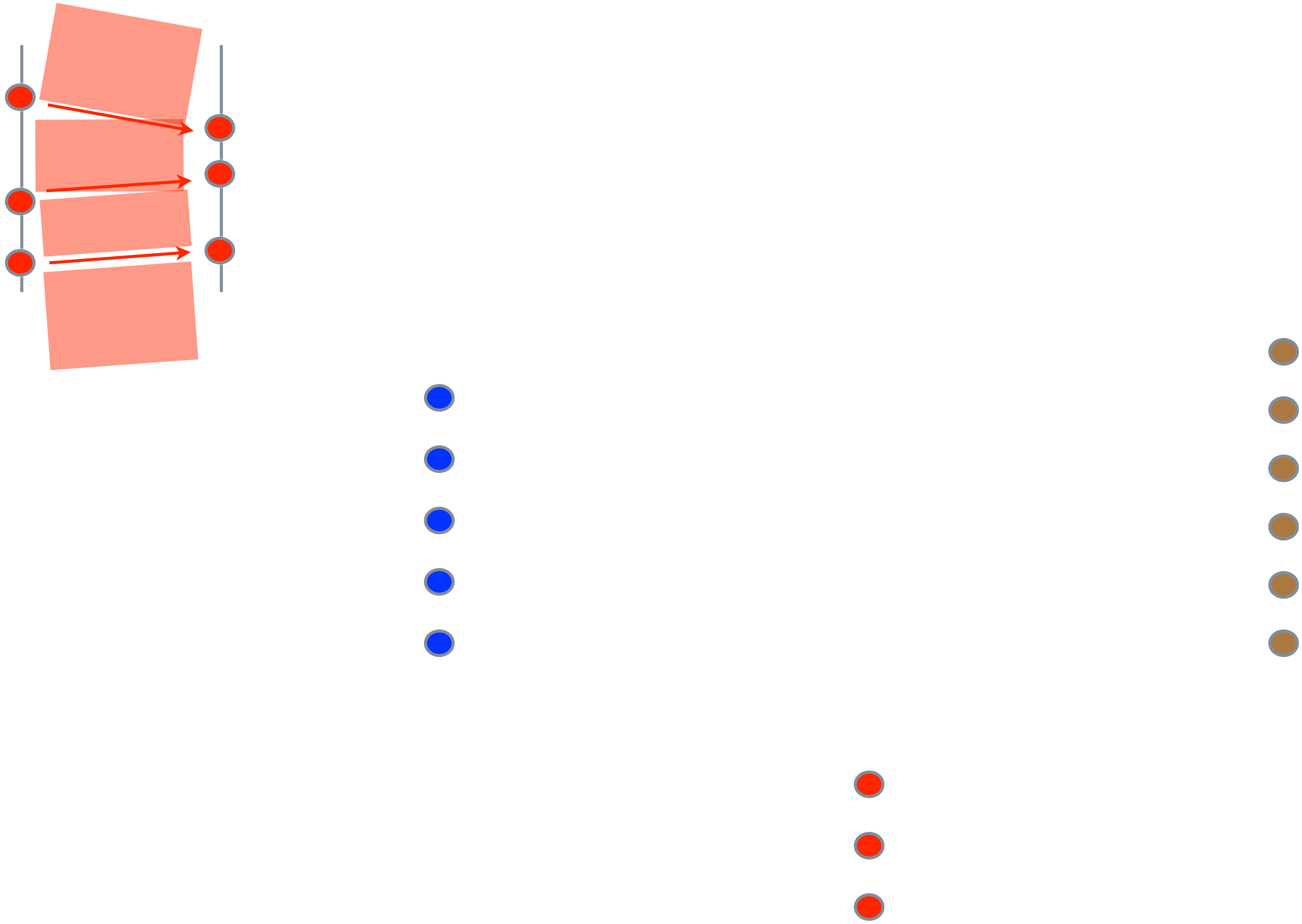

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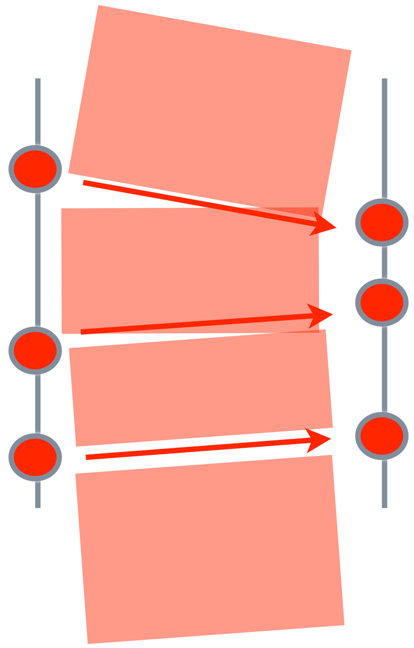

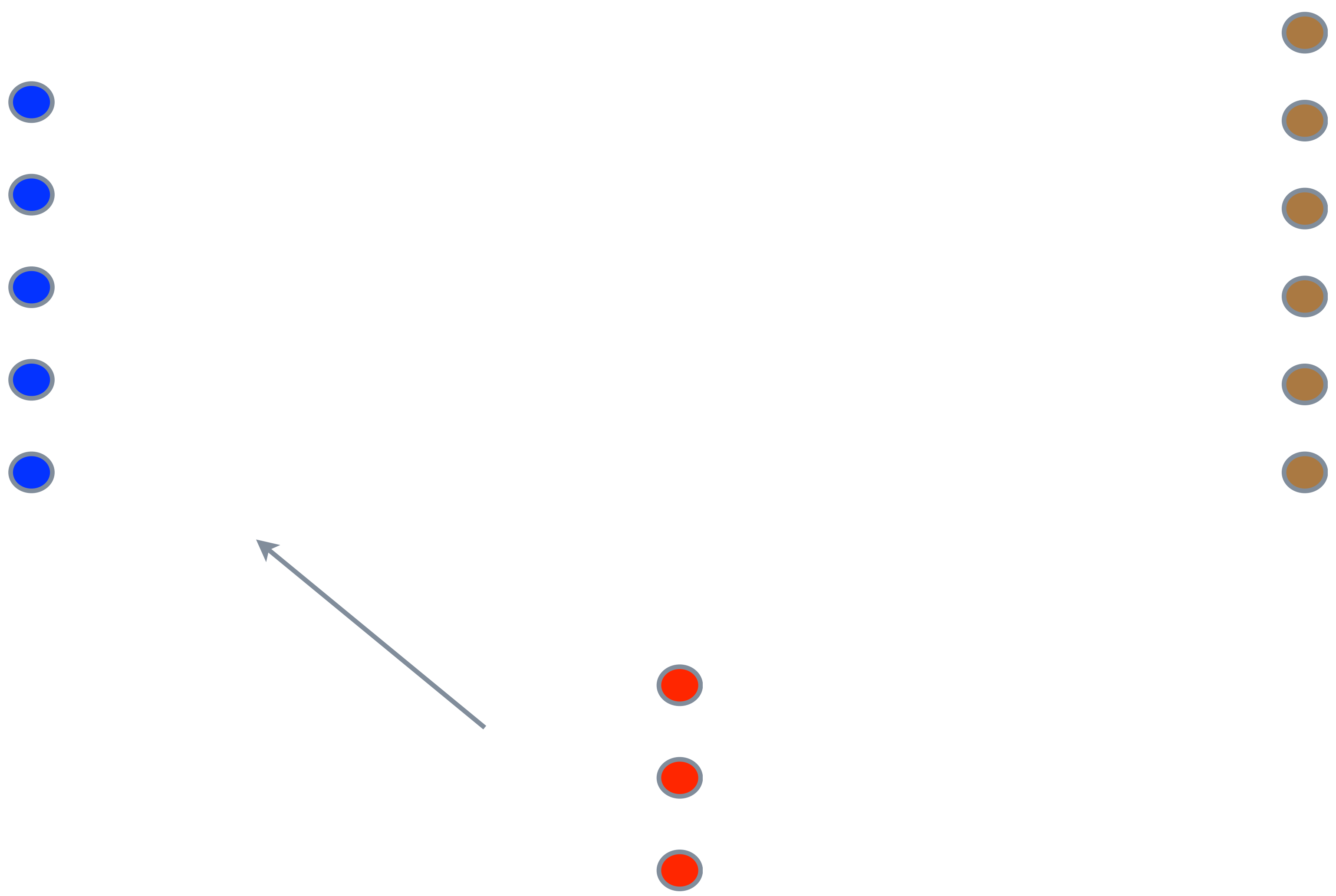

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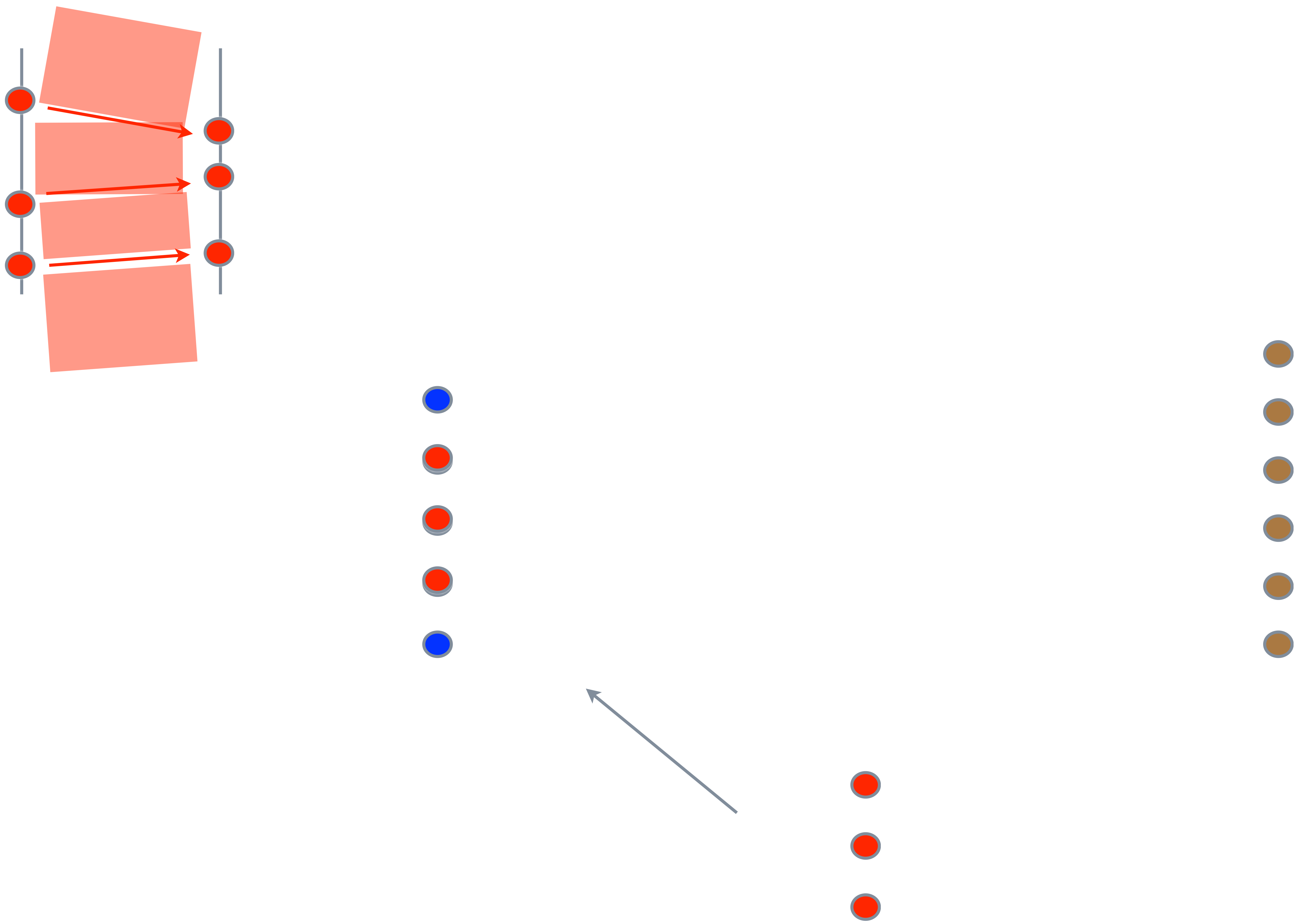

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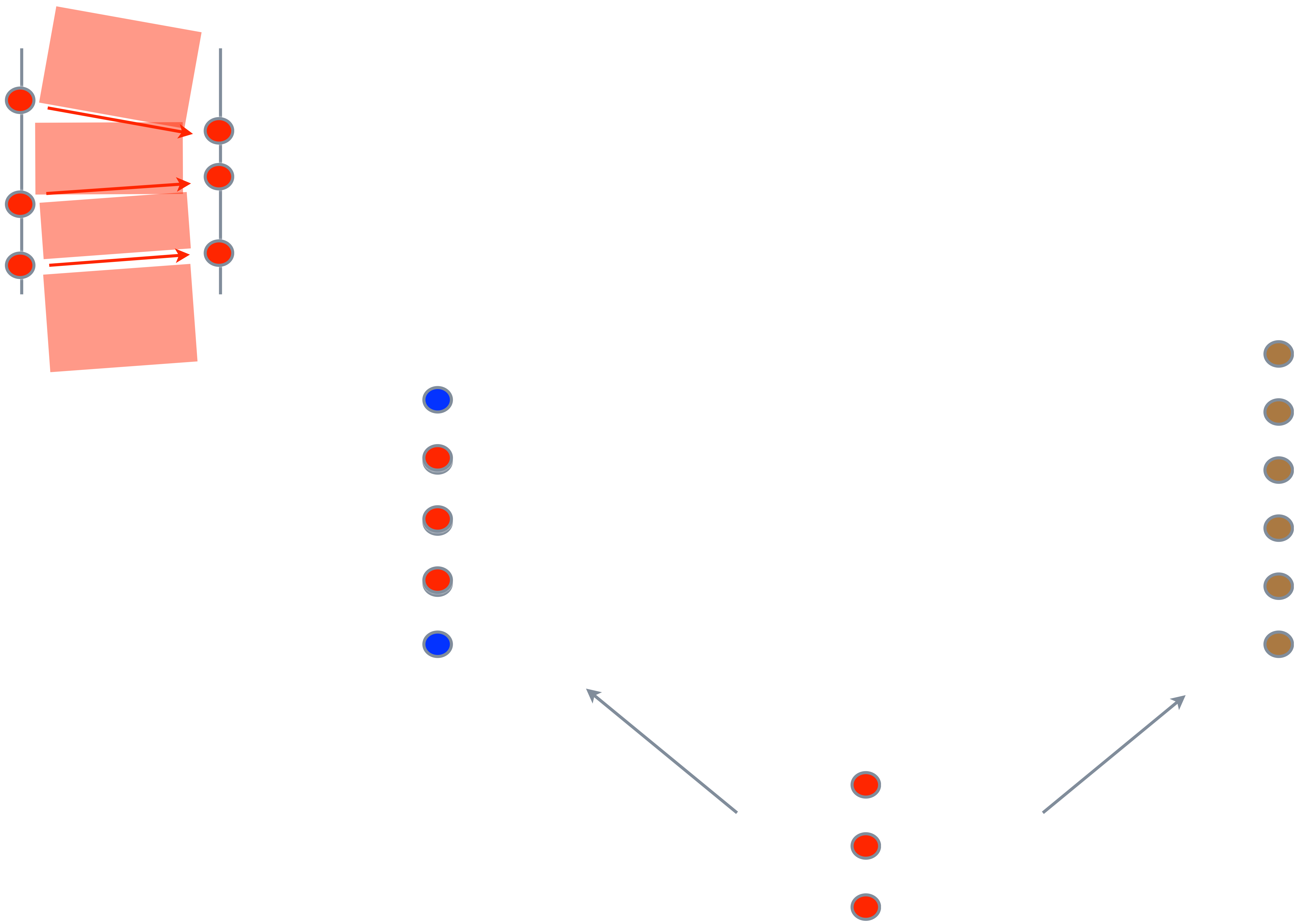

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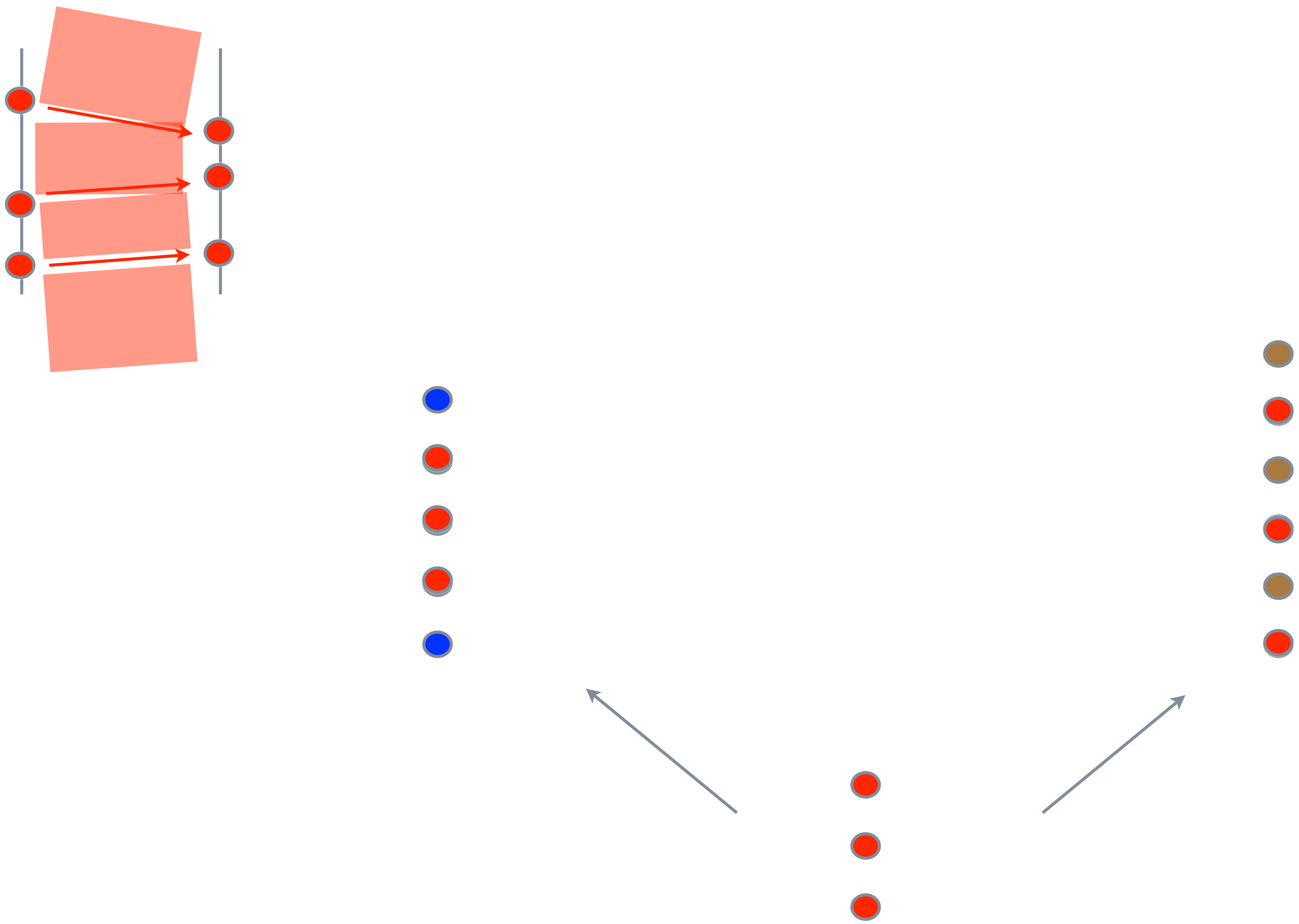

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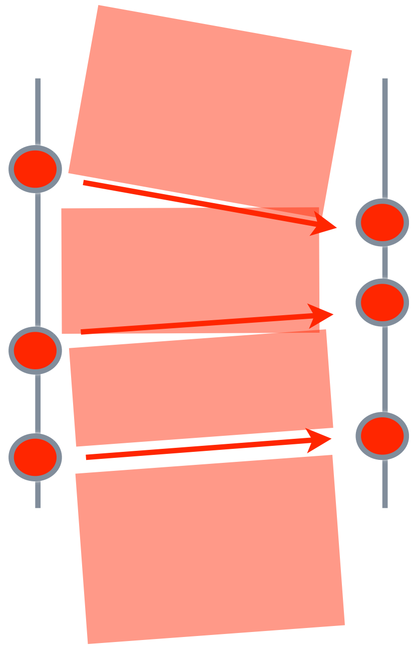

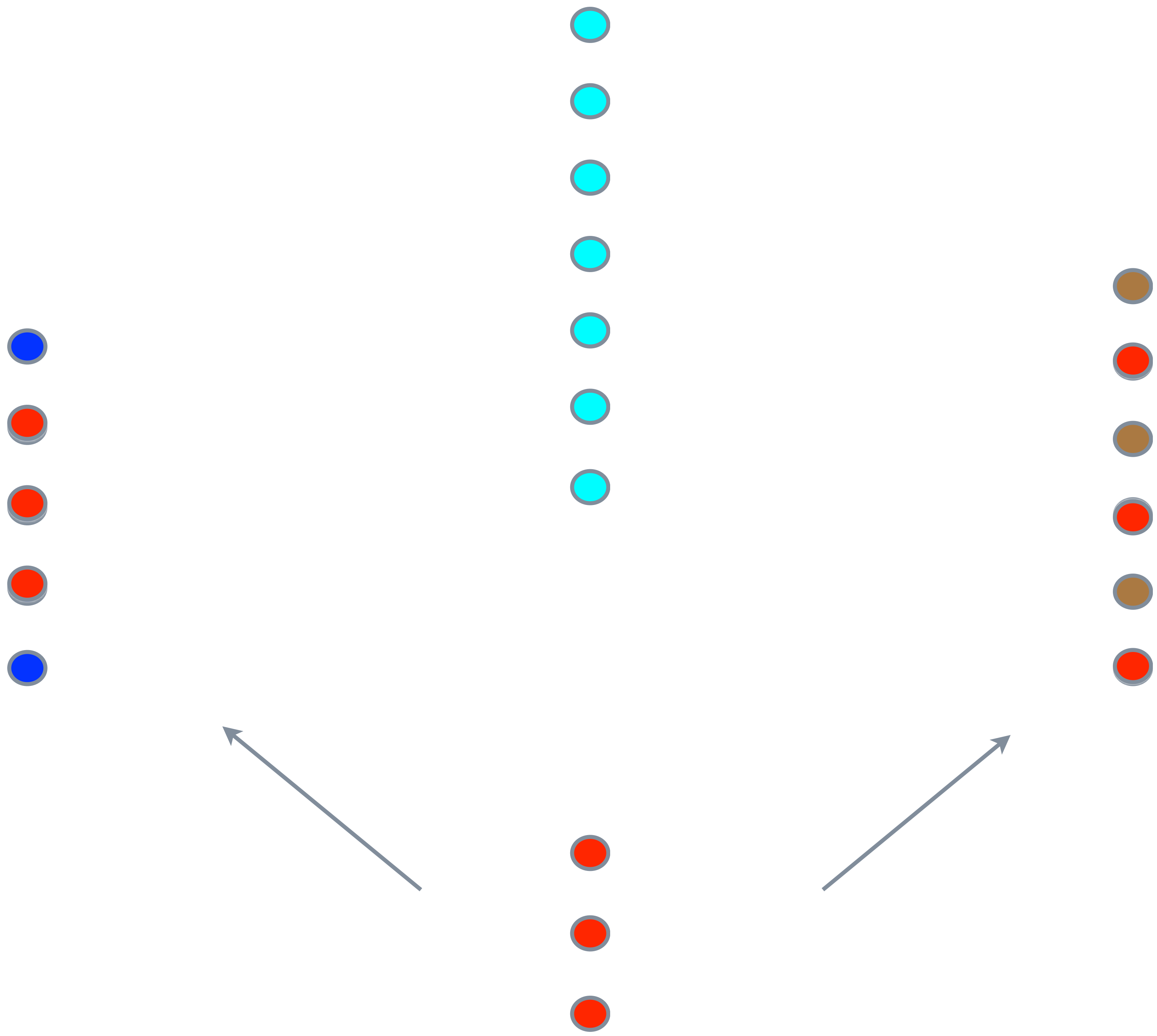

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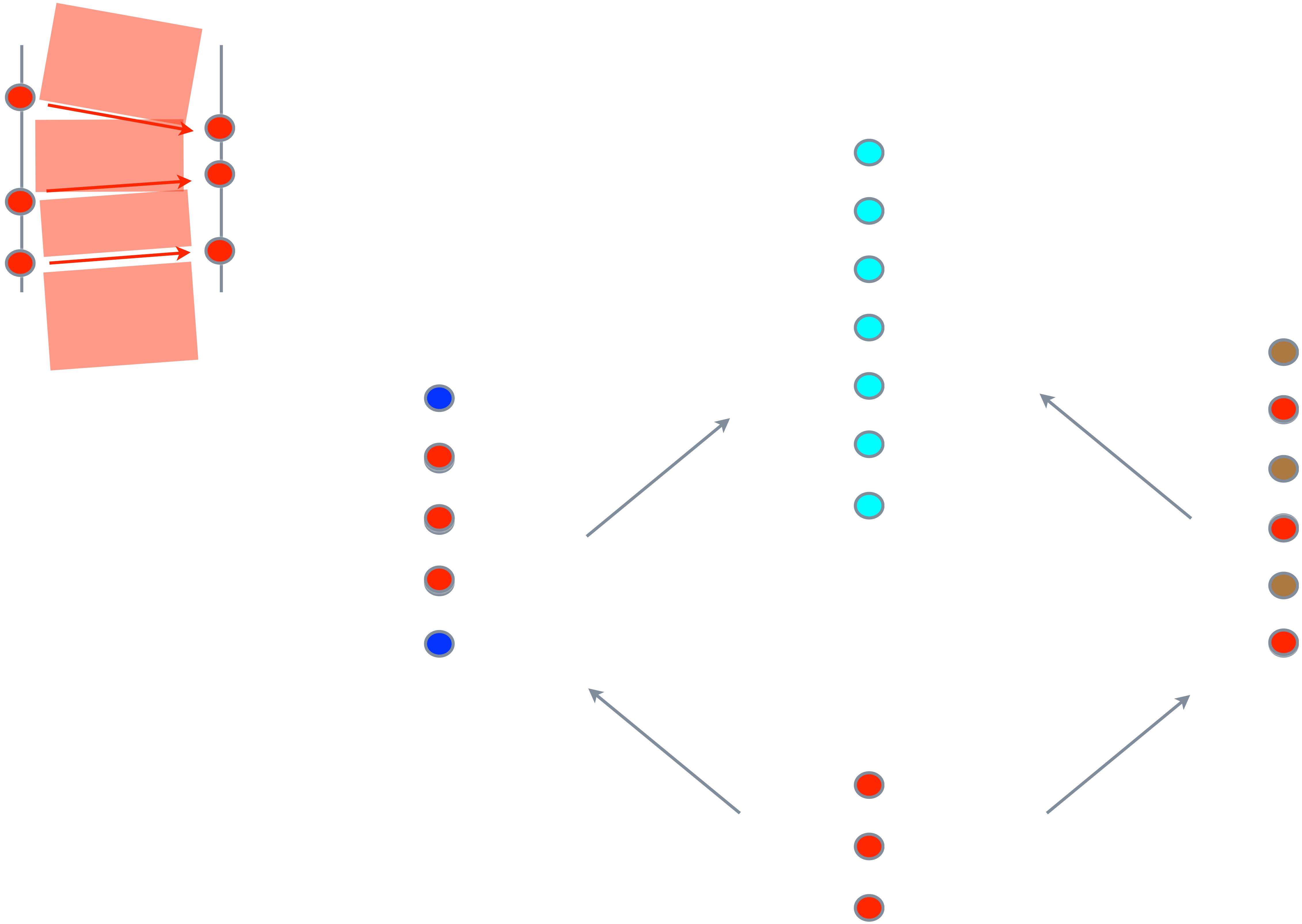

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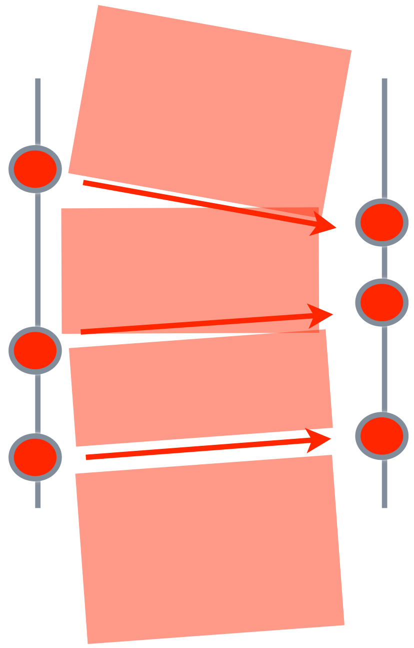

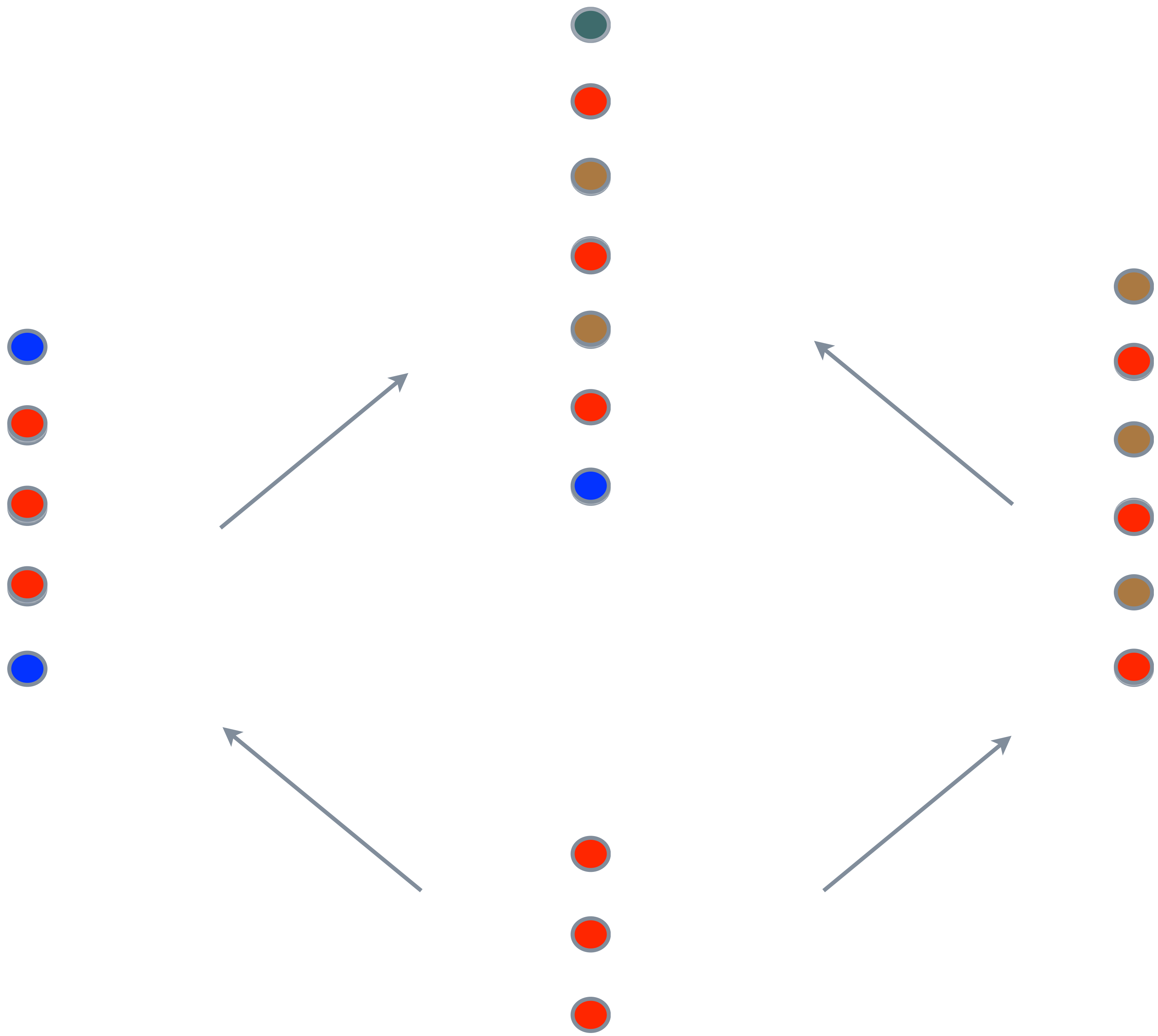

the question

For which data domains the standard problems are decidable?

the question

homogeneous

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classifications

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A classification of all homogeneous structures remains a great challenge.

Outline

- (un)ordered data Petri nets
- standard decision problems
- Petri nets with homogeneous data
- **undecidability**
- decidability via wqo

undecidability - example

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- data domain $(\mathbb{N}^2, =_1, =_2, =_{12})$

undecidability - example

- data domain $(\mathbb{N}^2, =_1, =_2, =_{12})$
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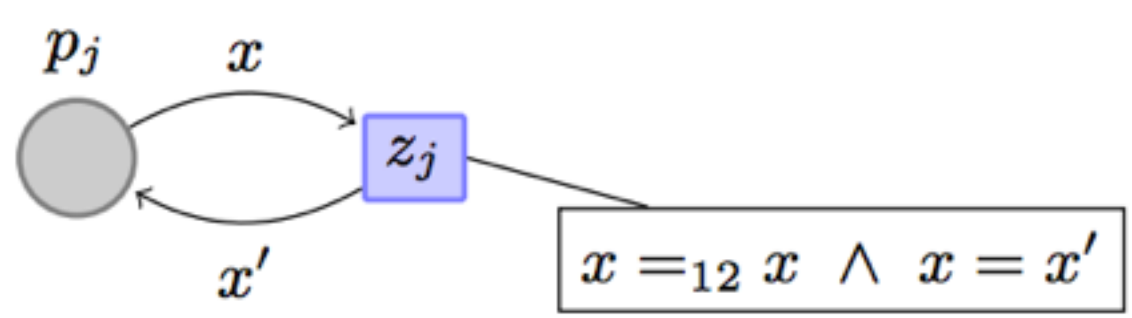

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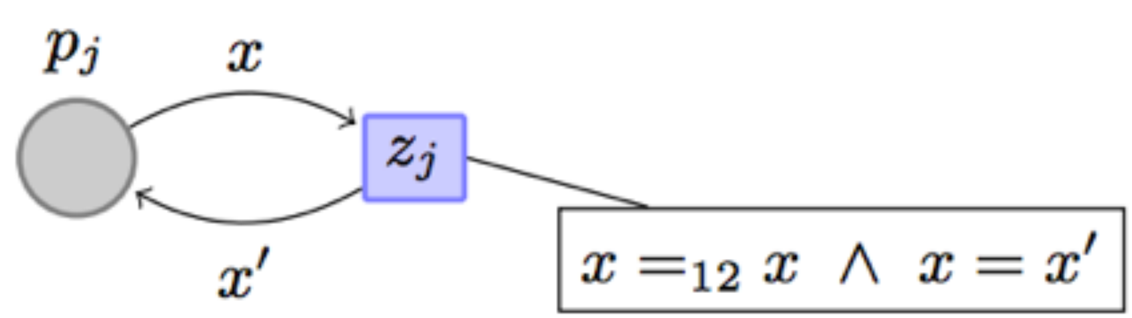

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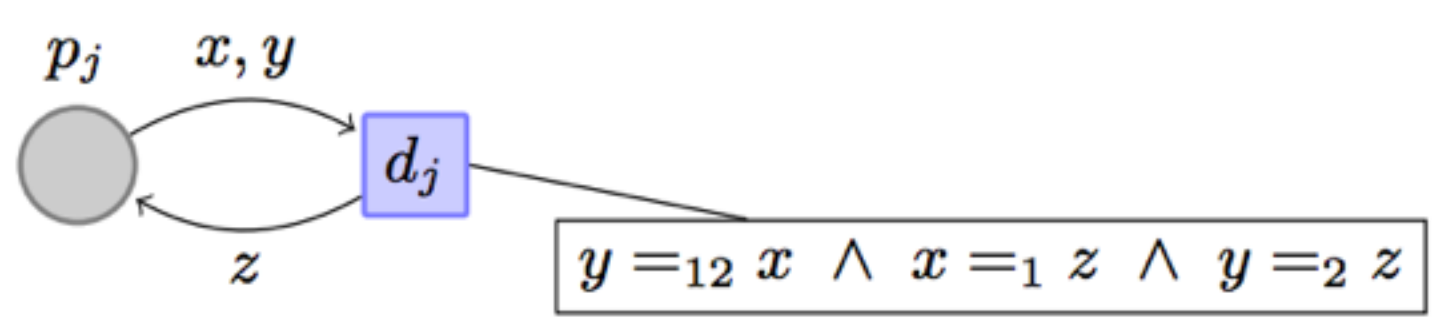

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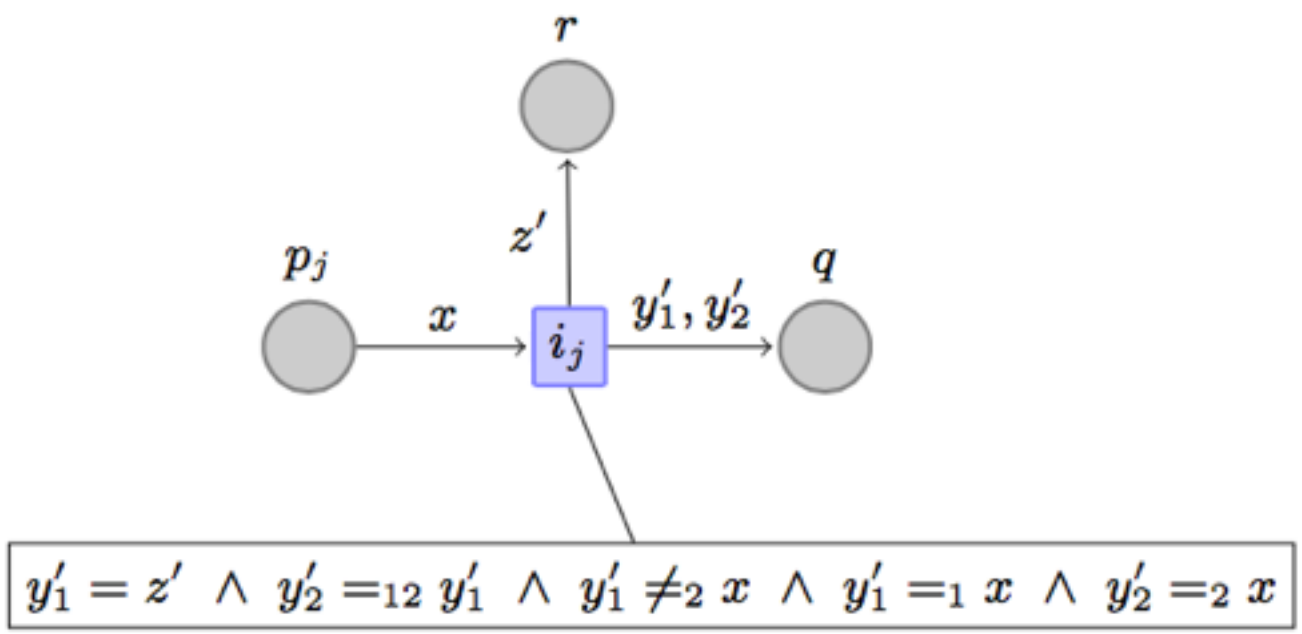

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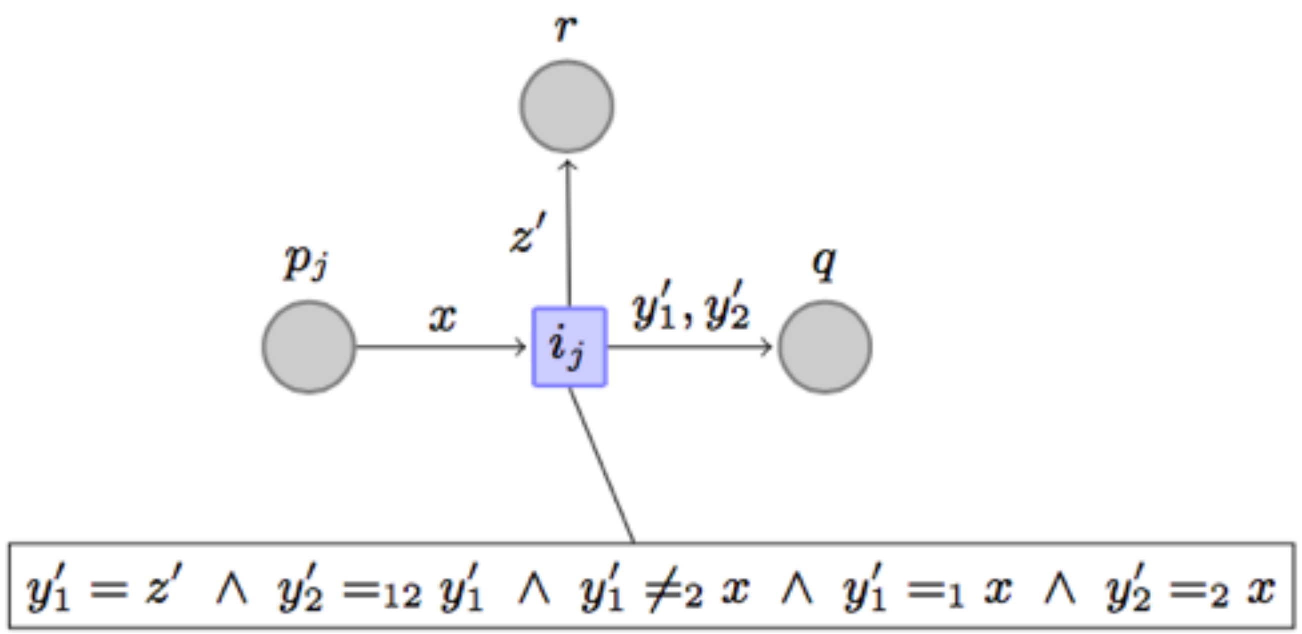

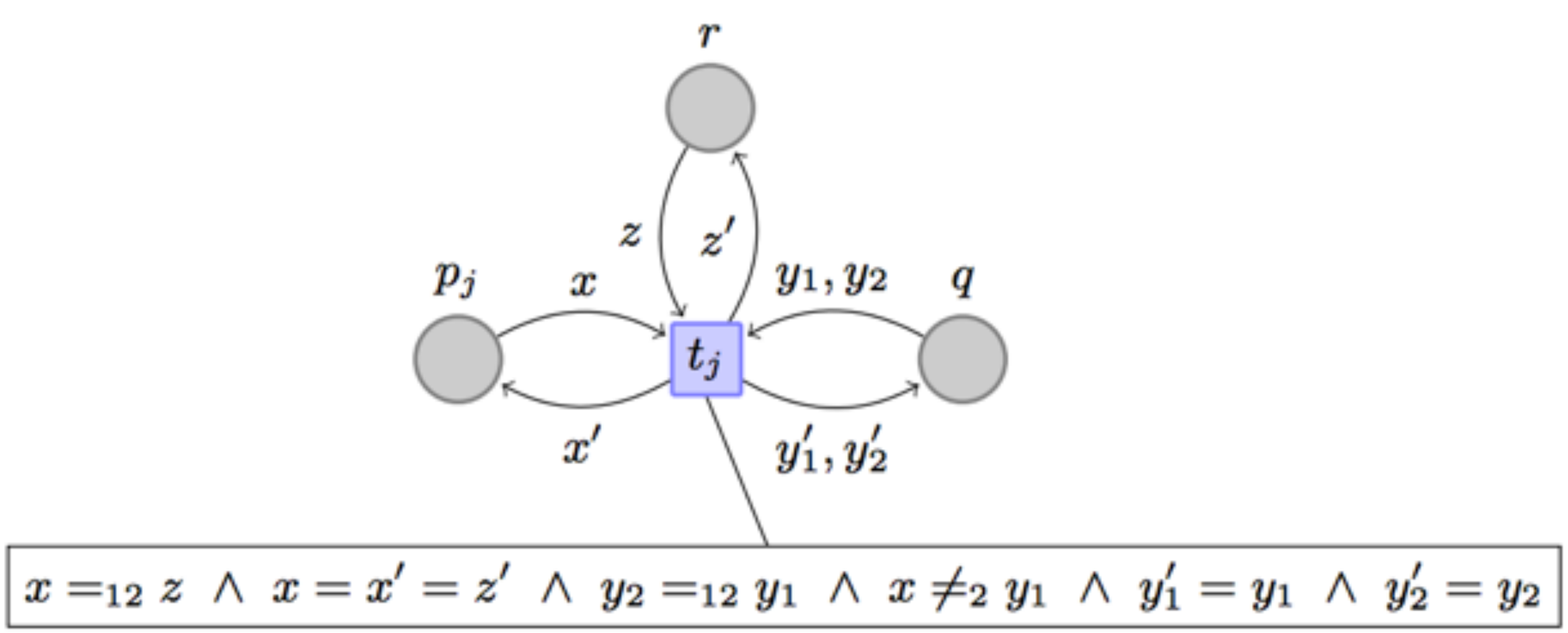

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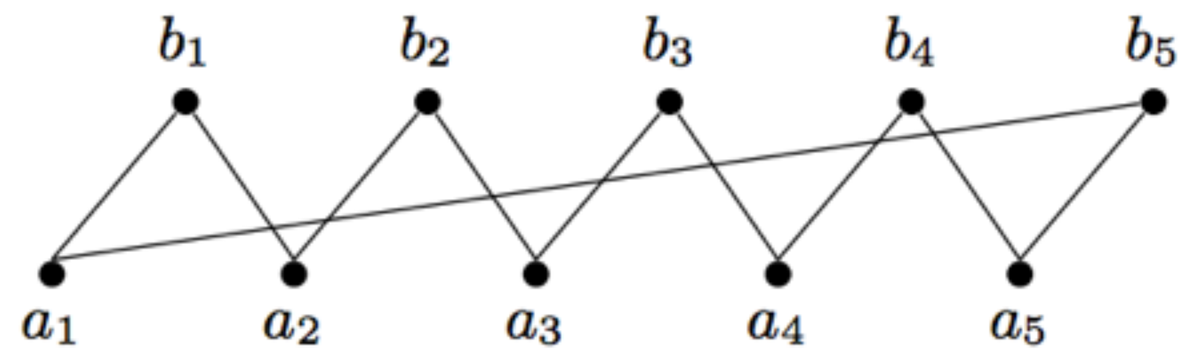

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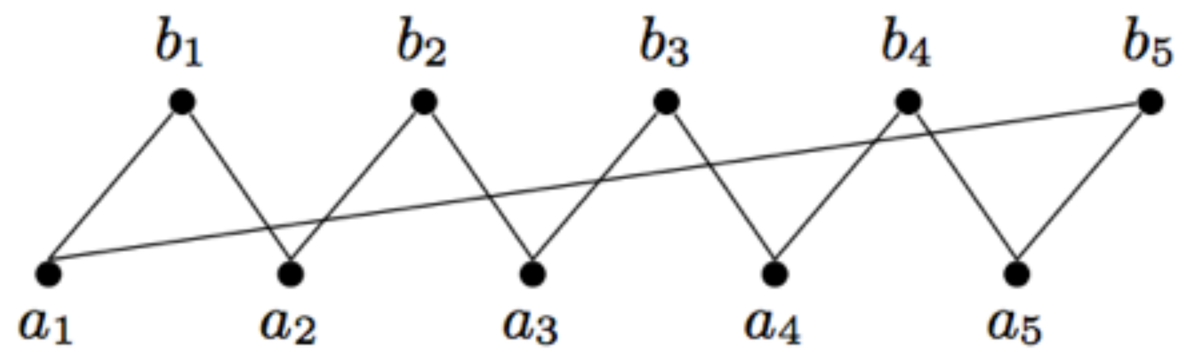

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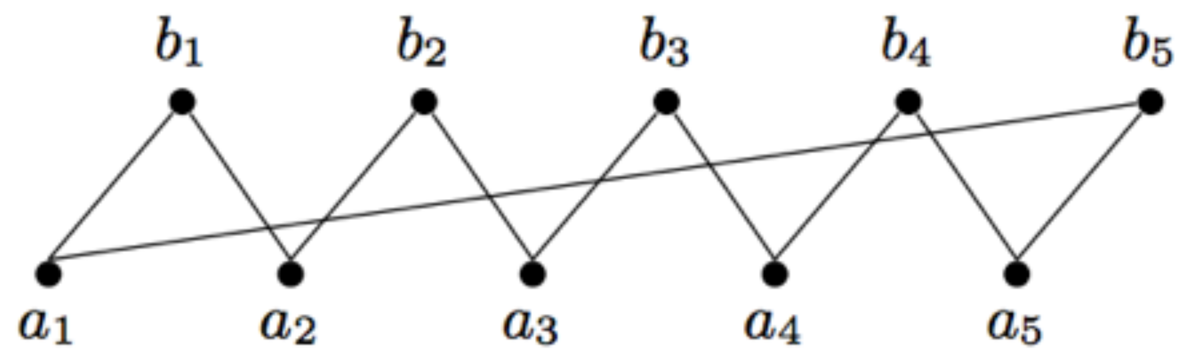

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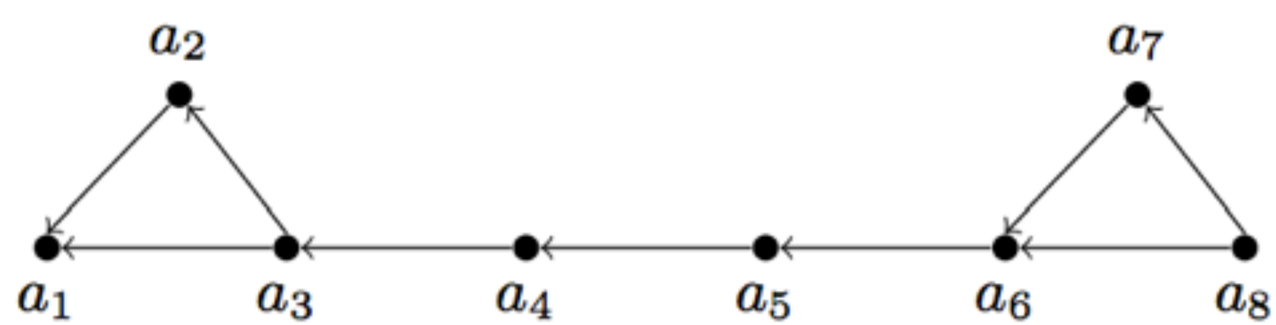

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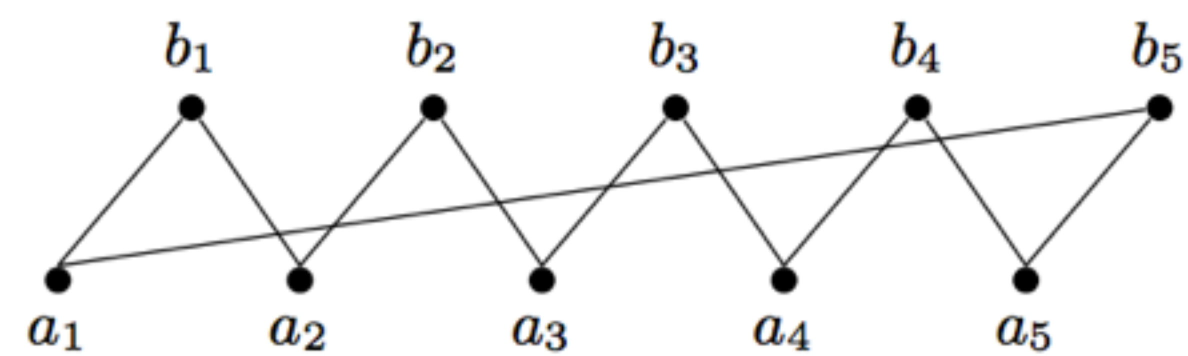

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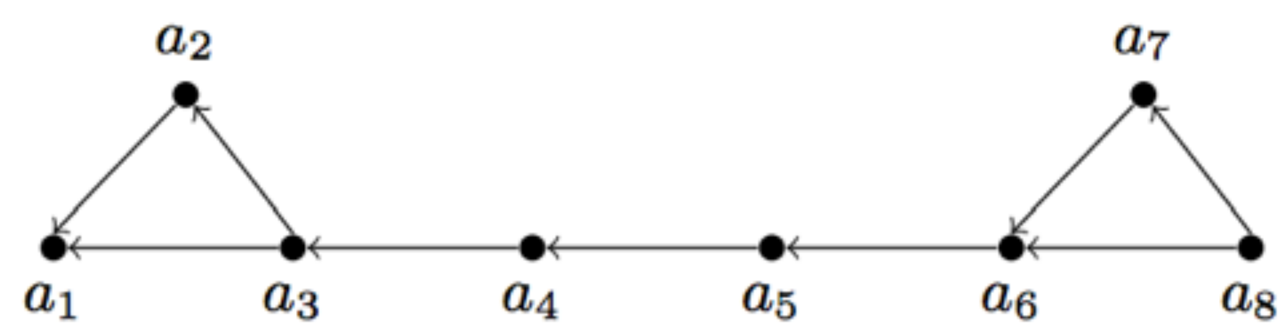

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$\mathbb{A}$ = random graph configurations = finite graphs labeled by elements of $M(\mathrm{P})$ no wqo!

not to be confused with Robertson-Seymour's Graph Minor Theorem!

decidability

Consider age($\mathbb{A}$) ordered by embeddings.
Recall that a configuration is a structure from age($\mathbb{A}$) labeled by elements of $M(\mathrm{P})$

Examples:

$\mathbb{A}$ = $(\mathbb{N}, =)$	configurations = $M(M(\mathrm{P}))$	wqo
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Theorem:

Let $\mathbb{A}$ be an effective homogeneous data domain such that configurations, ordered by embeddings, are a wqo.
Then all the standard problems are decidable.

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Proof: Using the framework of well-structured transition systems of [Finkel,Schnoebelen'01].

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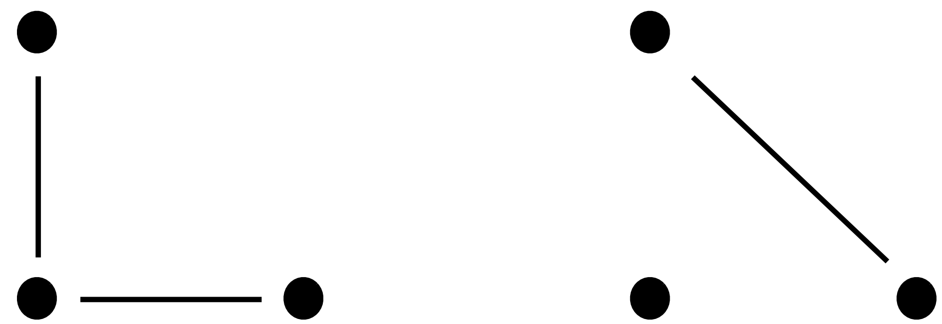

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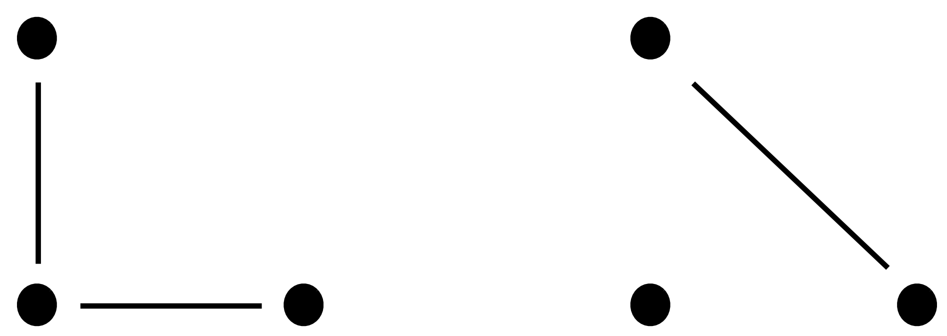

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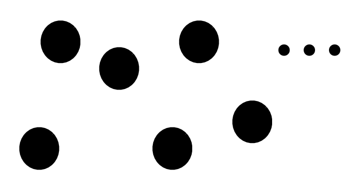

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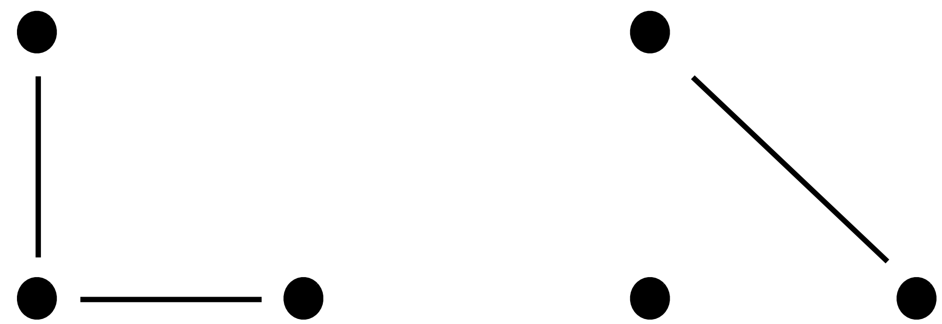

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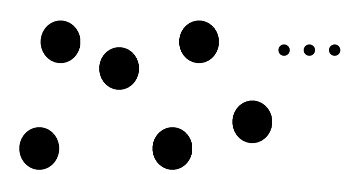

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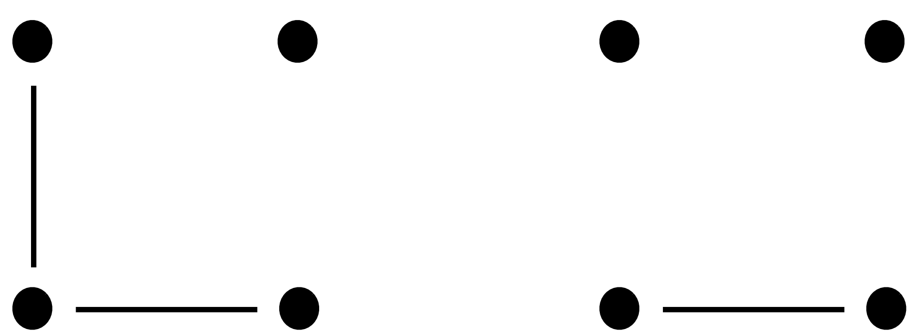

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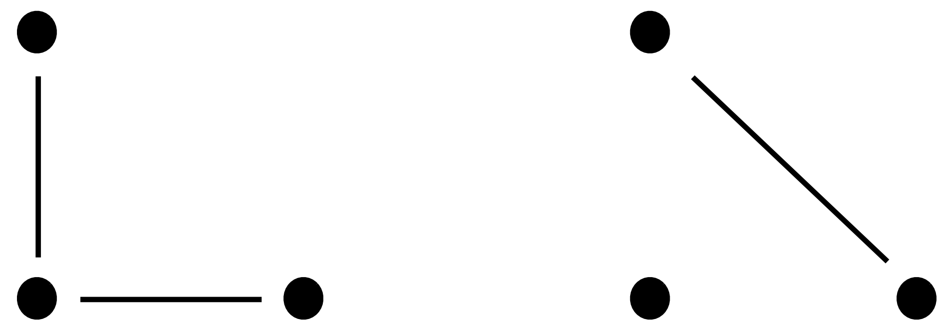

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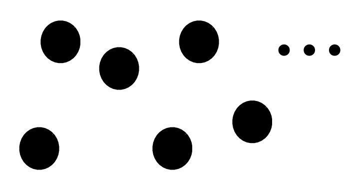

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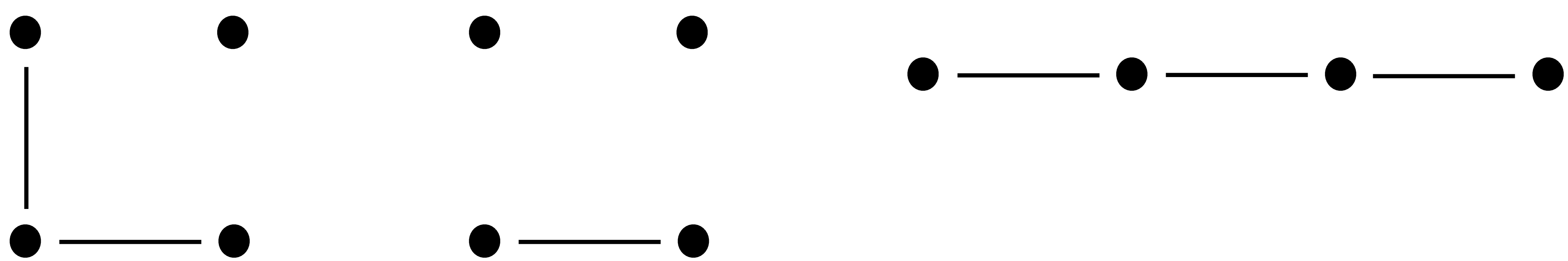

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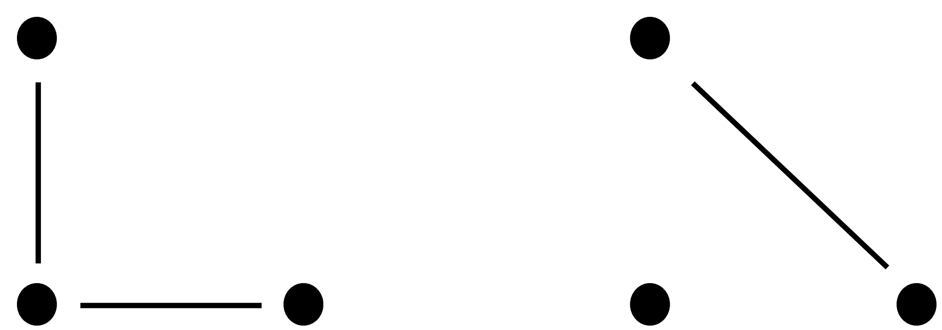

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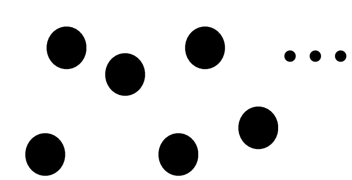

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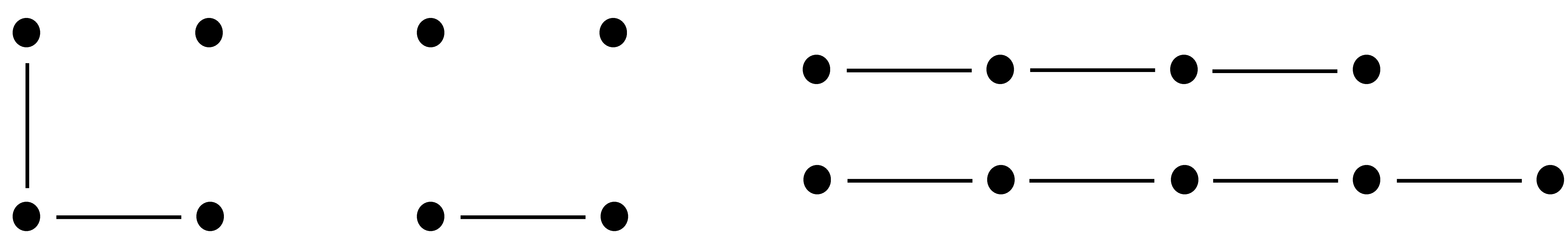

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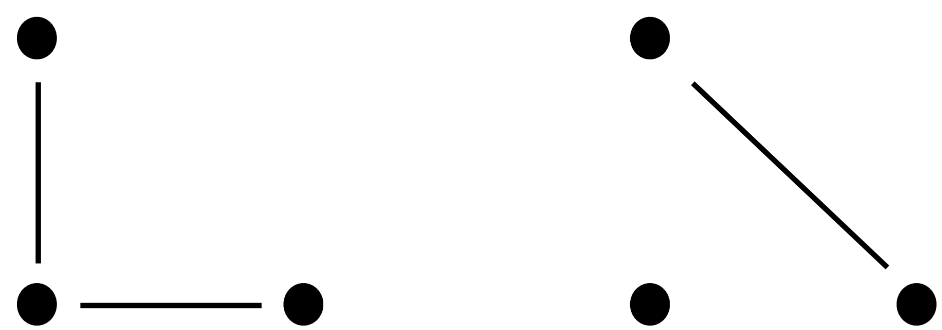

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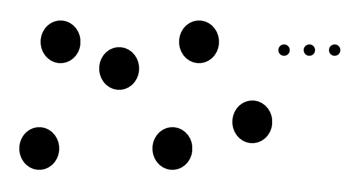

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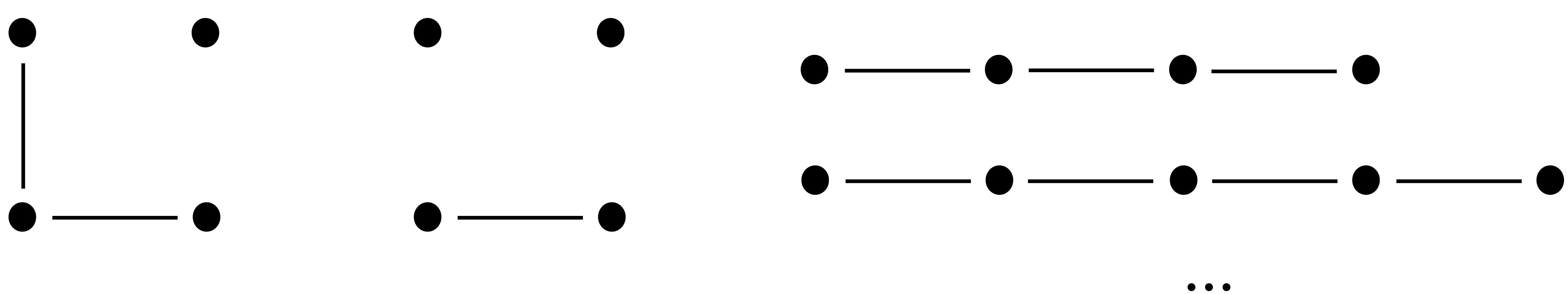

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