Algorithmic aspects of game theory

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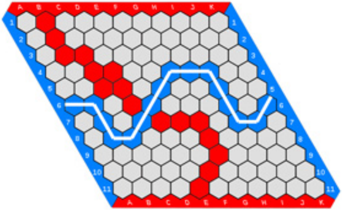
Determinacy of games

As noted by **Zermelo 1913**, in the game of chess, either

- **White** has a strategy to **win**, or
- **Black** has a strategy to **win**, or
- both players have strategies to force at least a **draw**.

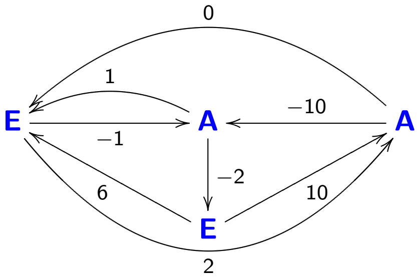
This holds for any **perfect-information** games (finitely winning).

But to **find a strategy** is another matter...



Games on graphs

A general model of a turn-based game.

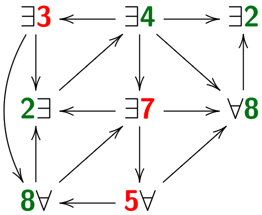


Idea: $\circ \xrightarrow{\mathbf{w}} \circ$ means that **Adam** pays **w** to **Eve**.

The result: asymptotic **mean payoff**.

The quest for an optimal strategy is in **NP ∩ co-NP**.

Parity games



Eve wants to visit **even** priorities infinitely often.

Adam wants to visit **odd** priorities infinitely often.

Maximal priority wins.

For this special case, a **quasi-polynomial** ($n^{\log n}$) algorithm was found in **2017**.

Complexity of games

Finding a **polynomial-time** algorithm for parity/mean-payoff games remains a big open challenge.

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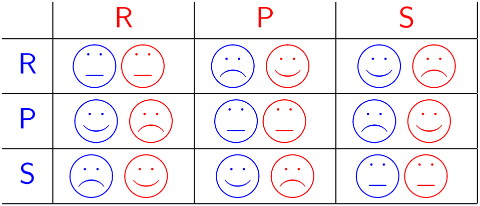
Nash equilibria

Rock, paper, scissors game



A related problem is to find a **mixed Nash equilibrium** (also in **NP** $\cap$ **co-NP**).

This problem is hard in a new complexity class **PPAD** (Constantinos Daskalakis, *Nevanlinna Prize* 2018).

Example of a homework

Barman–Client game

Barman and Client wear **blue** or **read** ties.

If they happen to wear both a **blue** tie, Client gets **one** drink.

If they happen to wear both a **read** tie, Client gets **two** drinks.

Otherwise Client pays Barman **x** and gets nothing.

	B	**R**
B	1	$-x$
R	$-x$	2

What should be **x**, so that the game would be **fair** ?