

Homework, the 1st series

Deadline: 2 April, 23:59.

Assume that a Turing machine M computes for input 1^n a code of a circuit C_n with n variables, using the space $\mathcal{O}(\log n)$ on its input tapes. Assume additionally that the circuit C_n is a *Boolean formula*, that is, its *Or* and *And* gates have fan-out at most 1. (This restriction does not apply to the input gates and the negated input gates.)

Prove that the language recognized by the sequence of circuits $(C_n)_{n \in \mathbb{N}}$, i.e.,

$$\{w \in (0 + 1)^* : C_{|w|}(w) = \mathbf{1}\}$$

is in the complexity class L .

Remark. We assume that the above M is an *off-line* Turing machine which, in addition to the input tape, has an *output tape*. On the output tape, the machine can write symbols in the empty cells and move the head to the right. The space restriction concerns only the working tapes; in particular it does not apply to the size of C_n . The code of a circuit should be understood as in the course; the gate identifiers can be chosen arbitrarily within the size limit $\mathcal{O}(\log n)$.

Hint. Show first that evaluation of Boolean formulas can be performed in logarithmic space.