Homework 3&4

Deadline: Wednesday, 4th June, 23:59

Problem 3

Show that the following problem is not in P. Given an alphabet A, two words $u, v \in A^*$ of equal length, and two binary relations $H, V \subseteq A^2$, decide if Player 1 has a winning strategy in the following game:

- the players take turns, Player 1 moves first;
- each move consists in choosing a word over A of length $|u|$, such that each two consecutive letters of the word are in relation H, and for all i, the i-th letter of the chosen word is in relation V with the i-th letter of the last word chosen by the opponent;
- in the first move Player 1 has no choice: he must play the word u;
- Player 1 wins if at some point word v is chosen (by him or the opponent).

Hint: $\mathrm{APS}\textsc{pace} = \textsc{ExpTime}$.

Problem 4

For a language $L \subseteq \{0,1\}^*$, let

$$B(L, r) = \{u \colon \exists v \in L\ d(u, v) \leq r\}$$

where $d(u, v)$ is the Hamming distance,

$$d(u,v) = \begin{cases} \left|\{i \colon u_i \neq v_i\}\right| & \text{if } |u| = |v|\,, \\ \infty & \text{if } |u| \neq |v|\,. \end{cases}$$

Show that for each $L \subseteq \{0,1\}^*$ and each $r \in \mathbb{N}$

(a) if $L \in \mathrm{RP}$ then $B(L, r) \in \mathrm{RP}$;

(b) if $L \in \textsc{co}\text{-RP}$ then $B(L, r) \in \textsc{co}\text{-RP}$.